Introduction to the Theory of Computing 2.

Exercise-set 3.
Solutions

1. a) $n_1 \cdot 1 + n_2 \cdot 2 + 5 \cdot 3 = 2(n_1 + n_2 + 5 - 1) \implies n_1 = 7$.
2. $10 \cdot 10 + (n - 10) \cdot 1 = 2(n - 1) \implies n = 92$
3. One of the degrees is 1. $d \cdot 9 + 92 \cdot 1 = 200 \implies d = 12$.
4. The tree has an even number of vertices.
5. $10(n - 1) = \binom{n}{2} - (n - 1) \implies n = 1$ or $n = 22$.
6. Necessary: $n - 1 = \binom{n}{2} - (n - 1) \implies n = 1$ or $n = 4$. Both are possible.
7. $k + (25 - k) \equiv 2 \cdot 24 \pmod{m} \implies m = 23$
8. a) no;
   b) yes.
9. A graph is a spanning tree and 3 more edges, each of which forms a cycle with the tree.
10. There is a cycle, of lenght at least 3.
11. The number of edges in a spanning forest is 17.
12. A degree one vertex in a spanning tree is like that.
13. a) Yes (2 triangles),
    b) No ($n - e + r = 2$).
14. No ($n - e + 2n = 2 \implies e = 3n - 2$, contradiction).
15. $n = 8,\ r = 10$.
16. $n = 20,\ r = 12,\ k \cdot r = 2e,\ n - e + r = 2 \implies k = 5$ (dodecahedron).
17. No, then $3n \leq e \leq 3n - 6$, contradiction.
18. a) If k vertices have degree 5 and $n - k$ more than 5, then $5k + 6(n - k) \leq 6n - 12 \implies k \geq 12$.
    b) No, e.g: icosahedron.
19. By contradiction, if both G and $\overline{G}$ are planar $\implies |E(G)|, |E(\overline{G})| \leq 13 \cdot 3 - 6$, but $|E(G)| + |E(\overline{G}|) = \binom{13}{2}$, contradiction.
20. At most 2: $e \leq 3n - 6$. Adding 2 edges is possible.
21. a) Then $|E| = 3(n - 1) > 3n - 6 \implies G$ cannot be planar.
    b) Otherwise it contained a spanning tree + a).
22. a) $|E(K_8) = 28 = (3 \cdot 8 - 6) + 10$, and each „additional" edge creates a new crossing with the „planar" ones.
    b) $|E(K_{4,4}) = 16 = (2 \cdot 8 - 4) + 4 \implies \exists \geq 4$ edge-crossings.
23. b), f) and k) are planar, the rest are nonplanar.
24. G cannot contain a subgraph homomorphic to K_5 or $K_{3,3}$.
25. Yes, G cannot contain a subgraph homomorphic to K_5 or $K_{3,3}$.
26. a) A nonplanar graph has at least 9 edges.
    b) The complement of a K_5 subgraph contains $K_{3,3}$.