

## Exercise-set 1. Solutions

1. a)  $26^5 - 26$ ,  
 b)  $7!/(2! \cdot 3!)$ ,  
 c)  $\binom{29}{5}$ ,  
 d)  $5 \cdot 25 \cdot 24 \cdot 23 \cdot 22$ ,  
 e)  $\binom{25}{4} = \binom{26}{5} - \binom{25}{5}$ ,  
 f)  $26^5 - 25^5$ ,  
 g)  $\binom{25}{5} + \binom{24}{4}$ ,  
 h)  $\binom{26}{9}$ ,  
 i)  $\sum_{k=0}^{26} \binom{26}{k} = 2^{26}$ .
2. a)  $5^{25}$ ,  
 b)  $\binom{24}{20}$ .
3. a)  $\binom{30}{6}$ ,  
 b)  $\binom{35}{6}$ ,  
 c)  $\binom{100}{4}$ .
4.  $\binom{30}{4}^{16}$ .
5. a)  $\binom{90}{20}$ ,  
 b)  $\binom{90}{20} + 19$ ,  
 c)  $\binom{89}{4} \cdot \binom{89}{13}$ ,  
 d)  $\binom{89}{5} + \binom{88}{3}$ .
6.  $\binom{26}{4}$ .
7. a)  $\binom{20}{4} \cdot ((\binom{20}{4} - 1) \cdot ((\binom{20}{4} - 2))$ .  
 b)  $\binom{20}{4} \cdot ((\binom{20}{4} - 1) \cdot ((\binom{20}{4} - 2) - \binom{19}{4}) \cdot ((\binom{19}{4} - 1) \cdot ((\binom{19}{4} - 2))$ .  
 c)  $3 \cdot \binom{19}{3} \cdot \binom{19}{4} \cdot ((\binom{19}{4} - 1))$ .
8.  $26 \cdot 25 \cdot 24 \cdot 23 \cdot 4^5$ .
9.  $6 \cdot \binom{5}{3} \cdot 11!/(3! \cdot (2!)^3)$ .
10.  $\binom{15}{3} \cdot \binom{12}{4} \cdot 2^4$ .
11.  $\binom{12}{4} \cdot \binom{8}{3} \cdot 24^5$ .
12.  $\binom{5}{5} \cdot \binom{5}{0} + \binom{5}{4} \cdot \binom{5}{1} + \binom{5}{3} \cdot \binom{5}{2} = 2^9$ .
13.  $26^6 + \binom{6}{1} \cdot 26^5 \cdot 10 + \binom{6}{2} \cdot 26^4 \cdot 10^2$ .
14.  $\binom{6}{3} \cdot 26^3 \cdot 10^3 - 4 \cdot 26 \cdot 10^3$ .
15.  $1 \cdot \binom{6}{2} \cdot 9^4 + 8 \cdot \binom{6}{3} \cdot 9^3$
16. a), b) The numbers of even and odd subsets are equal (see ex. 19/a).
17.  $2^{100}$ .
18. a) 100,  
 b) 100  
 c) 36.
19. a) 0,  
 b)  $\binom{100}{10}$ .