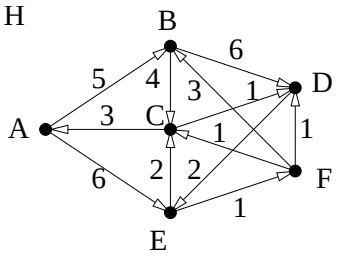
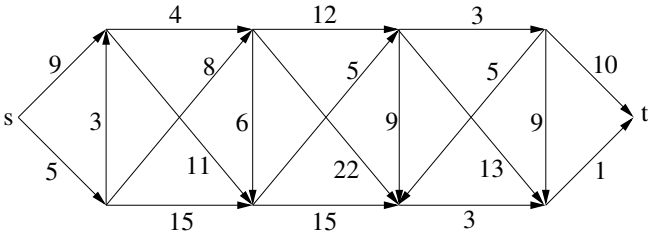
Introduction to the Theory of Computing 2.

Exercise-set 12.

1. a) Determine the shortest paths from vertex A to the other vertices in the first graph below using Dijkstra's algorithm.
   b) We decrease the weight of one edge by 1. For which edges will the distances from vertex A remain the same?
   c) We add the edge (B, E) to the graph. For which weights $l(B, E) \geq 0$ will the lengths of the shortest paths change?



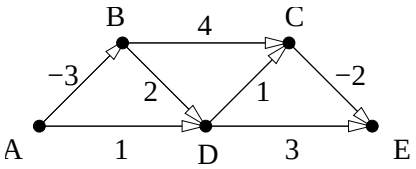


2. Determine the shortest paths from vertex s to vertex t in the second graph above using Dijkstra's algorithm.

3. Determine all the directed graphs with the minimum number of edges for which the table below gives the changes of the array containing the values $d(v)$(= the actual distance of v from v_1) in Dijkstra's algorithm. Determine the states of the array $p(v)$(= vertex preceding v) as well.

v_1	v_2	v_3	v_4	v_5	v_6
0	2	6	∞	∞	7
0	2	5	9	∞	6
0	2	5	6	9	6
0	2	5	6	8	6
0	2	5	6	7	6

4. Determine the shortest paths from vertex A to the other vertices in the graph below using Ford's algorithm.



5. We want to change money for our summer travel. At the exchange office n currencies are handled. For one unit of the jth type we have to pay r_{ij} units of the ith type. (E.g. if the jth is the US dollar, and the ith is the forint, then $r_{ij} = 275$ approximately.) Using the array r_{ij}, give an algorithm which determines the best exchange rate from our currency to all the others in $c \cdot n^3$ steps, if we suppose that there is no commission at the office. (We can change currency i to currency j in more than one steps.)

6. There is one edge of negative weight among the edges of the graph G, but the weights of all the other edges are positive. The graph doesn't contain a cycle of negative weight. Give an algorithm which determines the lengths of the shortest paths from $s \in V(G)$ to the other vertices in $c \cdot n^2$ steps.

7. In the software market there are programs converting n graphical formats to each other (back and forth). The price of the program converting the ith format to the jth format (back and forth) is p_{ij} and the running time of the same program is t_{ij} (if the program exists).
   a) Suggest a method for designing which programs to use if we want to convert each format to the format understood by our own graphics terminal as fast as possible. (The price does not matter.)
   b) Suggest a method for deciding which programs to buy so that with the help of those we can convert any format to any other format as cheap as possible. (Here the runtime does not matter).