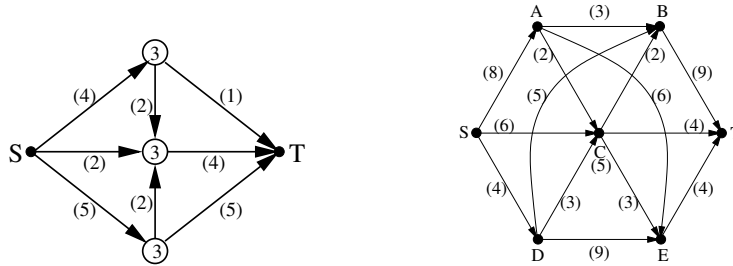
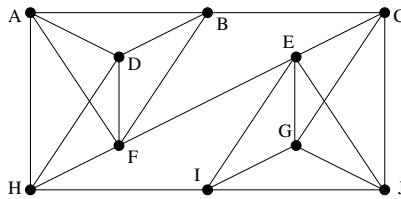


Exercise-set 10.

- In a network all the capacities are integers. Which of the statements below holds always?
 - Each maximum flow in the network has an integer value.
 - There is a maximum flow in the network which takes an integer value on each edge.
 - Each maximum flow in the network takes an integer value on each edge.
 - What about the same questions if we substitute „integer” for „even number” everywhere?
- Determine the value of a maximum flow in the networks with edge- and vertex capacities below, and prove that they are maximal.



- At most how many pairwise edge-disjoint and vertex-disjoint paths are there between the following points in the graph below:
 - B and I ,
 - A and J ,
 - B and H .



- (MT'12) The graph G on 15 vertices is constructed from three cycles, on 4, 5 and 6 vertices each, in such a way that each vertex of the 5-vertex cycle was connected (with one edge) to all the other vertices of the other two cycles. Let s be a vertex of the 4-vertex cycle, and t be a vertex of the 6-vertex cycle.
 - At most how many pairwise vertex-disjoint paths are there in G between s and t ?
 - At most how many pairwise edge-disjoint paths are there in G between s and t ?
- Determine the vertex- and edge connectivity numbers ($\kappa(G)$ and $\lambda(G)$) of the following graphs:
 - the graph consisting of the vertices and edges of a cube,
 - the complete bipartite graph $K_{m,n}$, where $m \geq n$,
 - the graph in Exercise 1.
- The vertices of an 18-vertex graph G can be divided into 3 classes of six vertices each, in such a way that 2 vertices are adjacent if and only if they are in different classes. Determine the largest integer k for which G is k -vertex-connected ($\kappa(G)$), and the largest integer l for which G is l -edge-connected ($\lambda(G)$).
- Show that a k -(vertex-)connected graph G on n vertices has at least $kn/2$ edges.
- Prove that an $n/2$ -(vertex-)connected graph on n vertices contains a Hamilton cycle.
- Construct a simple graph which is 2-vertex-connected, 3-edge-connected and has minimum degree 4.
- (MT'14) We connect two disjoint complete graphs on 5 vertices with 3 edges, in such a way that the resulting graph G is simple. Is it true in all cases that G is
 - 3-(vertex-)connected;
 - 3-edge-connected?
- (MT'17) A simple graph on 10 vertices has 40 edges. Determine the largest integer k for which G is surely k -vertex-connected.