Definition: A *walk* in a graph $G(V, E)$ is a sequence (of vertices and edges) $v_0, e_1, v_1, e_2, v_2, \ldots, v_{n-1}, e_n, v_n$, where $e_i = \{v_{i-1}, v_i\}$ for all $i = 1, 2, \ldots, n$.

Definition: A *trail* in a graph $G(V, E)$ is a walk where the edges cannot be repeated (but the vertices can).

Definition: A *circuit* in a graph $G(V, E)$ is a closed trail, i.e. one for which $v_0 = v_n$.

Definition: A *path* in a graph $G(V, E)$ is a walk where the vertices cannot be repeated (therefore the edges cannot be repeated either).

Definition: A *cycle* in a graph $G(V, E)$ is a closed path, i.e. one for which $v_0 = v_n$, but otherwise the vertices are all different.

Definition: A graph $G(V, E)$ is *connected*, if there is a path (/walk/trail) between any pair of its vertices.

The connectedness of a graph can be chechked with the BFS (breadth-first-search) algorithm, which finds (one of) the shortest path(s) from a vertex to all the other vertices.

Euler circuits and trails

Definition: An *Euler circuit/trail* in a graph $G(V, E)$ is a circuit/trail containing all the edges of G (therefore exactly once).

Theorem (Euler): A graph G contains an Euler circuit if and only if G is connected and all the vertices have even degrees.

Theorem (Euler): A graph G contains an Euler trail if and only if G is connected and there are 0 or 2 vertices of odd degree.

Hamilton cycles and paths

Definition: A *Hamilton cycle/path* in a graph $G(V, E)$ is a cycle/path containing all the vertices of G (therefore exactly once).

Necessary conditions

Proposition: If a graph contains a Hamilton cycle and we delete k of its vertices, then the remaining graph can have at most k components, for $k = 1, 2, \ldots, n/2$.

Proposition: If a graph contains a Hamilton path and we delete k of its vertices, then the remaining graph can have at most $k + 1$ components, for $k = 1, 2, \ldots, n/2$.

Sufficient conditions

Theorem (Ore): If G is a simple graph on $n \geq 3$ vertices, and $\deg(x) + \deg(y) \geq n$ holds for any pair of nonadjacent vertices x, y, then G contains a Hamilton cycle.

Theorem (Dirac): If G is a simple graph on $n \geq 3$ vertices, and $\deg(v) \geq n/2$ holds for each vertex v, then G contains a Hamilton cycle.