1. We call a vector $v$ in $\mathbb{R}^5$ *semisymmetric* if both the sum of its first and last coordinates and the sum of its second and fourth coordinates are equal to the third (middle) coordinate. (E.g. the vector $(1, 3, 8, 5, 7)^T$ is a semisymmetric vector.) Determine the dimension of the subspace $V$ of $\mathbb{R}^5$ consisting of semisymmetric vectors. (For the solution you don’t need to show that $V$ is in fact a subspace.)

2. Solve the following system of linear equations for each value of the parameter $p$.

$$
\begin{align*}
  x_1 + 2x_2 + 4x_3 & = 6 \\
  2x_1 + 6x_2 + p \cdot x_3 & = 12 \\
  x_1 + 8x_2 + 6x_3 & = 6
\end{align*}
$$

3. Let $A$ be the matrix below. Evaluate the determinant of $A \cdot A^T$

$$
A = \begin{pmatrix}
  2 & 4 & 2 \\
  3 & 4 & 1 \\
  2 & 1 & 3
\end{pmatrix}
$$

4. The matrix below is the inverse of the matrix $B$. Decide whether the matrix $B^2$ has an inverse, and if yes then determine it (where $B^2$ denotes the matrix $B \cdot B$).

$$
B^{-1} = \begin{pmatrix}
  3 & 5 \\
  2 & 3
\end{pmatrix}
$$

5. All we know of the $5 \times 5$ matrix $C$ is that it has exactly 3 non-zero entries. Determine all possible values of the rank of $C$.

6. * The determinant of the $10 \times 10$ matrix $D$ is 0. Moreover, no matter how we change one entry of $D$, the determinant of the matrix obtained is always 0 as well. Show that the rank of $D$ is at most 8.