Introduction to the Theory of Computing 1.
First Retake of the Second Midterm Test
2022. December 12.

1. We call a vector $\underline{v}$ in $\mathbf{R}^4$ *average* if the coordinates of it, starting from the third one, are the averages of the previous two coordinates. (E.g. the vector $(1, 5, 3, 4)^T$ is an average vector.) Determine the dimension of the subspace V of $\mathbf{R}^4$ consisting of the average vectors. (For the solution you don't need to show that V is in fact a subspace.)

2. Determine the number of solutions of the system of equations below for each value of the parameters p, q. (You don't have to determine the solutions themselves.)

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 8 \\ x_1 + 3x_2 + p \cdot x_3 &= 9 \\ x_1 + q \cdot x_2 + 4x_3 &= p + 4 \end{aligned}$$

3. Evaluate the determinant below *using the original definition.* (So don't use any properties of the determinant, or theorems about it during the solution, but determine the value using the definition only.)

$$\begin{pmatrix} 7 & 0 & 0 & 6 \\ 0 & 0 & 5 & 0 \\ 0 & 4 & 0 & 0 \\ 3 & 0 & 0 & 2 \end{pmatrix}$$

4. Let A be the matrix below. Evaluate the determinant of A^2 (where A^2 denotes the matrix $A \cdot A$).

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 3 & 4 & 5 \end{pmatrix}$$

5. Let A be the matrix below.
   a) Determine the matrix $A^2 - 2A$.
   b) Decide whether the matrix $A - I$ has an inverse, and if yes then determine it.

$$A = \begin{pmatrix} 14 & 6 \\ -28 & -12 \end{pmatrix}$$

6. * Let A be an arbitrary 2×2 matrix. Show that we can add two rows and two columns to A in such a way that the 4×4 matrix obtained is invertible.

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts.

Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1, A2 or A3) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. To obtain a signature you have to achieve at least 24 points on each of the two midterm tests.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used. The problem marked with an * is supposed to be more difficult.