

# Introduction to the Theory of Computing I.

## Second Midterm Test

2022. December 2.

1. Determine for which values of the parameter  $p$  the system of equations below is consistent. If it has solutions, then determine all of them.

$$\begin{aligned} -x_1 + 3x_2 - 2x_3 + x_4 &= 0 \\ 4x_1 - 12x_2 + 13x_3 - 4x_4 &= 15 \\ 3x_1 - 9x_2 + 2x_3 + (p-3)x_4 &= p^2 - 2p - 12 \end{aligned}$$

2. Somebody evaluated the following determinant four times one after another, and he obtained four different values: 1020, 2022, 3024 and 4026. Finally he checked it with a computer and it turned out that one of the four values was in fact correct. Which one?

$$\begin{vmatrix} -6 & 2 & -6 & -6 \\ 12 & 14 & 18 & 12 \\ -6 & 14 & -16 & -8 \\ 6 & 4 & 22 & 10 \end{vmatrix}$$

3. Compute the matrix  $A^{11}$  for the matrix  $A$  below. ( $A^{11}$  is the product with 11 factors, each of whose is  $A$ .)

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 4 & -6 & 4 \\ 3 & -6 & 5 \end{pmatrix}$$

4. Decide whether the matrix  $A$  above is invertible or not, and if yes, then determine  $A^{-1}$ .
5. We prepare the  $3 \times 4$  matrix  $B$  from the matrix  $A$  below by adding a new row to it which is the sum of the rows of  $A$ . After that we prepare the  $3 \times 5$  matrix  $C$  from the obtained matrix  $B$  by adding a new column to it which is the sum of the columns of  $B$ . Finally we prepare the  $4 \times 5$  matrix  $D$  from the matrix  $C$  by adding a new row to it which is the sum of the rows of  $C$ . Determine the rank of the matrix  $D$ .

$$A = \begin{pmatrix} 2 & -5 & 1 & 3 \\ -4 & 10 & -2 & 4 \end{pmatrix}$$

6. \* Let  $U$  be a subspace of  $\mathbf{R}^4$  and  $V$  a subspace of  $\mathbf{R}^6$ , and suppose that  $\dim U = 3$  and  $\dim V = 4$ . Let the set  $W \subseteq \mathbf{R}^{10}$  consist of those vectors in  $\mathbf{R}^{10}$  whose first 4 coordinates form a column vector in  $U$  and last 6 coordinates form a column vector in  $V$ . Show that  $W$  is a subspace of  $\mathbf{R}^{10}$  and determine the value of  $\dim W$ .

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts.

Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1, A2 or A3) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. To obtain a signature you have to achieve at least 24 points on each of the two midterm tests.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used. The problem marked with an \* is supposed to be more difficult.