Introduction to the Theory of Computing 1.
Re-retake of the First Midterm Test
2022. December 19.

1. Determine the last 4 digits of $3^{100\,000}$.

2. Use the algorithm we learnt to determine the remainder we get when we divide 3^{73} by 77. (So don't use any other methods for the solution, the goal is to demonstrate how the algorithm works.)

3. A ball (whose shape is a regular sphere) is rolling on a slope with a flat surface. In the moment when the ball touches the slope in the point $P(2, 5, -1)$ its center is in the point $C(16, 1, 7)$. Does the plane of the slope pass through the origin?

4. In $\mathbf{R}^4$ let $\underline{u} = (1, 5, 4, 8)^T$, $\underline{v} = (0, 0, 1, 3)^T$ and $\underline{w} = (0, 2, 1, 1)^T$. Determine span$\{\underline{u}, \underline{v}, \underline{w}\}$, the subspace spanned by $\underline{u}, \underline{v}$ and $\underline{w}$. (I. e. give an equation satisfied by the coordinates of the vectors in span$\{\underline{u}, \underline{v}, \underline{w}\}$.)

5. In $\mathbf{R}^5$ let $\underline{a} = (2, 3, 5, 7, 11)^T$, $\underline{b} = (2, 3, 5, 7, 13)^T$ and $\underline{c} = (2, 3, 5, 7, 17)^T$. Decide whether the vectors $\underline{a}, \underline{b}, \underline{c}$ are linearly independent or not.

6. * The remainder of an integer n when divided by 100 is 1 larger than the remainder of n when divided by 73. How much bigger is the remainder of n when divided by 7300 than the remainder of n when divided by 73?

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts. Write your name on every sheet you work on, and write your Neptun code on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. The problem marked with an * is supposed to be more difficult.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators (or similar devices) cannot be used.