

Introduction to the Theory of Computing 1.

First Retake of the First Midterm Test

2022. November 4.

1. What can the remainder of an integer be when divided by 273 if 57 times that integer gives a remainder of 99 when divided by 273?
2. Determine the remainder we get if we divide 2^{1032} by 70.
3. Determine the common point of the lines e and f , where e has system of equations $\frac{x-1}{3} = \frac{y-2}{7} = \frac{z-3}{11}$ and f is perpendicular to the plane of equation $2x + 5y + 4z = 7$ and passes through the point $P(-2, -6, 2)$.
4. Let $V \subseteq \mathbf{R}^6$ be the set of vectors which have an equal number of positive and negative coordinates. (E.g. $\underline{v} = (2, 0, 5, -1, -3, 0)^T$ is in V , but $\underline{w} = (3, -2, 6, 0, 2, -7)^T$ is not.) Decide whether V forms a subspace in \mathbf{R}^6 or not.
5. We only know of the vectors $\underline{a}, \underline{b}, \underline{c}, \underline{d} \in \mathbf{R}^3$ that $\text{span}\{\underline{a}, \underline{b}, \underline{c}\} = \text{span}\{\underline{a}, \underline{b}, \underline{d}\}$. Show by example that in this case the vectors $\underline{b}, \underline{c}, \underline{d}$ can be linearly independent and by another example that they can be linearly dependent as well. (You have to prove that the examples are correct.)
6. * Determine the value of

$$\max_{n \in \mathbf{Z}^+} \text{gcd}(21n + 6, 6n + 4),$$

that is, determine the largest possible value of the g.c.d. of $21n + 6$ and $6n + 4$ where n can be any positive integer.

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts.

Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1, A2 or A3) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. To obtain a signature you have to achieve at least 24 points on each of the two midterm tests.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used. The problem marked with an * is supposed to be more difficult.