Introduction to the Theory of Computing 1.
First Midterm Test
October 18, 2022

1. How many integers are there between 1 and 10 000 whose last two digits in the decimal system are 34 and whose last digit in the hexadecimal system is 'E'? (The hexadecimal system is the numeral system of base 16, and 'E' denotes the digit '14' in it.)

2. Decide if the number below is an integer or not.

$$\frac{5 \cdot 279^{961} + 5}{1400}$$

3. Using the Euclidean algorithm determine all the integers between 0 and 301 which when multiplied by 222 give 34 as a remainder when divided by 302. (The task includes documenting the computations as well.)

4. Does the plane through the intersection point of the lines $e: \ \frac{x}{7} = \frac{15-2y}{12} = 3 - z$ and $f: \ \frac{x-1}{2} = 2 - y, \ z = 1$ perpendicular to e pass through the point $P(9, -1, 1)$?

5. Let the set V consist of the vectors in $\mathbf{R}^5$ for which it holds that the sum of their first three coordinates is greater than or equal to the sum of their last two coordinates. Decide whether V forms a subspace in $\mathbf{R}^5$ or not.

6. * We know that the vectors $\underline{a}$, $\underline{b}, \underline{c}$ and $\underline{d}$ are linearly independent in $\mathbb{R}^n$, but the vectors $\underline{a}+\underline{b}+\underline{c}$, $\underline{a}-\underline{b}-3\underline{d}$, $\underline{a}+\underline{c}+5\underline{d}$ are linearly dependent. Does it imply that $\underline{d} \in \text{span}\{\underline{a}, \underline{b}, \underline{c}\}$ holds?

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts.

Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1, A2 or A3) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. To obtain a signature you have to achieve at least 24 points on each of the two midterm tests.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used. The problem marked with an * is supposed to be more difficult.