

Introduction to the Theory of Computing 1.

Second Retake of the Second Midterm Test

2021. December 20.

1. Solve the following system of linear equations.

$$\begin{aligned}x_1 + 2x_2 + 3x_3 + 9x_4 &= 6 \\2x_1 + 3x_2 + 5x_3 + 6x_4 &= 10 \\x_1 + 5x_2 + 2x_3 + 5x_4 &= 8 \\4x_1 + 9x_2 + 9x_3 + 8x_4 &= 21\end{aligned}$$

2. Evaluate the determinant of the matrix below for all the values of the parameters p, q .

$$\begin{pmatrix} 1 & 2 & 3 & 6 \\ 2 & 4 & 8 & 11 \\ 4 & q & 3 & 9 \\ 0 & 0 & 2 & p \end{pmatrix}$$

3. Let A be the matrix below. Does there exist a matrix X for which $A \cdot X$ is the 2×2 identity matrix? If the answer is yes, then determine one such matrix.

$$A = \begin{pmatrix} 2 & 6 & 8 \\ 3 & 10 & 0 \end{pmatrix}$$

4. Decide whether the matrix A in exercise 2. is invertible for the values $p = 0, q = 9$ or not. If yes, then determine the third entry in the first column of the inverse
5. Show that every 4×4 matrix A of rank 4 is the sum of 2 matrices of rank 2 (that is, there exist matrices B, C of rank 2 for which $B + C = A$ holds).
6. * A 3×3 matrix has two 0's on its main diagonal, and all the other entries of it are odd integers. Show that the matrix has an inverse.

Please work on stapled sheets only, and submit all of them at the end of the midterm. Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1 or A2) on the first page.

You have 90 minutes to work on the problems; each of them is worth 10 points. The problem marked with an * is supposed to be more difficult.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators (or similar devices) cannot be used.