

# Introduction to the Theory of Computing 1.

## First Retake of the Second Midterm Test

2021. December 13.

1. Decide whether the matrix  $A$  below is invertible or not. If yes, determine  $A^{-1}$  and the determinant of  $A^{-1}$ .

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 5 \\ 5 & 6 & 0 \end{pmatrix}$$

2. Determine for which values of the parameter  $p$  the system of equations below is consistent. If it has solutions, then determine all of them.

$$\begin{aligned} px_1 + px_2 + px_3 &= p \\ 2x_1 + 6x_2 + 10x_3 &= 6 \\ 3x_1 + 3x_2 + (p+3)x_3 &= 3 \end{aligned}$$

3. Evaluate the determinant of the matrix below for all the values of the parameter  $q$ .

$$\begin{pmatrix} 0 & q & 1 & 3 \\ 2 & q & 1 & 4 \\ 3 & 6 & 3 & 9 \\ 1 & 0 & 0 & 7 \end{pmatrix}$$

4. Determine the rank of the matrix above for all the values of the parameter  $q$ .
5. Let  $A$  be a  $5 \times 5$  matrix of rank 5. Show that  $A$  is the sum of 5 matrices of rank 1 (that is, there exist matrices  $A_1, A_2, \dots, A_5$  all of rank 1 for which  $A_1 + A_2 + \dots + A_5 = A$  holds).
6. \* Is it true that if all  $2 \times 2$  submatrices of a  $5 \times 5$  matrix  $A$  are invertible then  $A$  is invertible as well?

Please work on stapled sheets only, and submit all of them at the end of the midterm. Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1 or A2) on the first page.

You have 90 minutes to work on the problems; each of them is worth 10 points. The problem marked with an \* is supposed to be more difficult.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators (or similar devices) cannot be used.

Please turn over for the first midterm.