

# Introduction to the Theory of Computing 1.

## Second Retake of the First Midterm Test

2021. December 20.

1. Determine the remainder of  $n$  when divided by 309 if we know that  $178n - 1$  is divisible by 309.
2. Use the algorithm we learnt to determine the last two digits of  $4^{49}$ . (So don't use any other methods, the goal is to demonstrate how the algorithm works.)
3. Determine the remainder we get when we divide  $701^{701701}$  by 99.
4. Determine the equation of the plane which contains the points  $A(3, 4, 2)$  and  $B(4, 2, -1)$  and is perpendicular to the plane of equation  $4x + y + 9z = 1$ .
5. Is there a basis of  $\mathbf{R}^4$  containing the vectors  $\underline{a} = (1, 0, 0, 0)^T$ ,  $\underline{b} = (8, 5, 2, 0)^T$  and  $\underline{c} = (9, 6, 3, 0)^T$ ? If the answer is yes, then determine such a basis; if it is no, then prove it.
6. \* We know that from a set of five vectors in  $\mathbf{R}^4$  we can select 4 vectors which form a basis of  $\mathbf{R}^4$  in exactly one way. Show that in this case one of the vectors is the zero vector.

Please work on stapled sheets only, and submit all of them at the end of the midterm. Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1 or A2) on the first page.

You have 90 minutes to work on the problems; each of them is worth 10 points. The problem marked with an \* is supposed to be more difficult.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators (or similar devices) cannot be used.

Please turn over for the second midterm.