

Introduction to the Theory of Computing 1.

First Retake of the First Midterm Test

2021. December 13.

1. What is the remainder of $3^{147} + 70^{147}$ when divided by 73?
2. How many integers are there between 1 and 2021 which give a remainder of 18 when divided by 63 and a remainder of 34 when divided by 91?
3. The system of equations of line e is $x = y = \frac{z-3}{2}$, and the system of equations of line f is $\frac{x-3}{2} = y = z$. Decide whether e and f are coplanar or not (that is, whether there exists a plane containing both lines or not).
4. Decide whether those vectors in \mathbf{R}^4 which have two equal coordinates form a subspace in \mathbf{R}^4 or not.
5. Let the vectors $\underline{a}, \underline{b}, \underline{c}, \underline{d} \in \mathbf{R}^4$ be linearly independent. Does it imply that the set of vectors $\underline{a}, \underline{b}, \underline{c}, \underline{a} + \underline{b} + \underline{c} + \underline{d}$ is a basis of \mathbf{R}^4 ?
6. * Is it true that if $a^{40} \not\equiv b^{40} \pmod{100}$ holds for the integers a, b then $a^{40}b^{40} \not\equiv 1 \pmod{100}$?

Please work on stapled sheets only, and submit all of them at the end of the midterm. Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1 or A2) on the first page.

You have 90 minutes to work on the problems; each of them is worth 10 points. The problem marked with an * is supposed to be more difficult.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators (or similar devices) cannot be used.

Please turn over for the second midterm.