

# Introduction to the Theory of Computing 1.

## First Midterm Test

October 28, 2021

1. What is the remainder of 106 times the integer  $n$  when divided by 271, if this remainder is 1 more than the remainder of  $n$  itself when divided by 271?
2. What is the remainder of  $2021^{2021} - 2021^{101}$  when divided by 600?
3. Determine the value of the parameter  $p$  and the equation of the plane  $S$  if we know that  $S$  contains the points  $A(1, 2, 2)$  and  $B(3, 4, 1)$  and it is perpendicular to the line  $e$  with system of equations  $\frac{2x-7}{12} = \frac{8-y}{5} = \frac{z}{p}$ .
4. Let  $\underline{u}$  and  $\underline{v}$  be the vectors in  $\mathbf{R}^3$  below. Determine all those elements of the subspace spanned by  $\underline{u}$  and  $\underline{v}$  whose second coordinate is by 2, and third coordinate is by 3 larger than the first one.

$$\underline{u} = (3, 10, 1)^T, \quad \underline{v} = (1, -6, -2)^T$$

5. Determine those values of the parameter  $p$  for which the vectors  $\underline{a} = (2, 2, 2, 2)^T$ ,  $\underline{b} = (1, 2, 1, 1)^T$ ,  $\underline{c} = (1, 1, 3, 1)^T$  and  $\underline{d} = (1, 1, 1, p)^T$  form a basis in  $\mathbf{R}^4$ .
6. \* Determine all the positive primes  $p$  for which for all integers  $x$  and all positive primes  $q$  which are different from  $p$  the congruence  $x^2 \equiv 1 \pmod{p \cdot q}$  implies that either  $x \equiv 1 \pmod{p \cdot q}$  or  $x \equiv -1 \pmod{p \cdot q}$  holds.

Please work on stapled sheets only, and submit all of them at the end of the midterm. Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1 or A2) on the first page.

You have 90 minutes to work on the problems, each of them is worth 10 points. To obtain a signature you have to achieve at least 18 points on each of the two midterm tests, and the total points from the two midterms should be at least 48.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used. The problem marked with an \* is supposed to be more difficult.