

Introduction to the Theory of Computing I.

First retake of the midterm

14 December, 2020

1. What is the remainder when we divide 2020^{2021} by 1011?
2. We multiply an integer n by 17 and the result gives the remainder 23 when it is divided by 65. What can be the remainder when we divide n by 130?
3. Determine the parametric equations of the line which is obtained as the intersection of the planes given by the equations $x + 3y + 2z = 7$ and $4x + 6y + 5z = 10$.
4. Let V be the set of those vectors in \mathbb{R}^4 whose coordinates (ordered by position) form an arithmetic progression, i.e. the difference of the consecutive coordinates is the same (if we subtract the coordinate with the smaller index from the coordinate with the greater index). Decide if V is a subspace of \mathbb{R}^4 or not.
5. The vectors \underline{a} , \underline{b} and \underline{c} are linearly independent in \mathbb{R}^n . Does this imply that the vectors $2\underline{a}$, $\underline{a} + \underline{b}$ and $\underline{a} + \underline{c}$ are linearly independent?
- 6*. Determine all the integers $1 \leq a \leq 100$ for which $a^{21} \equiv 1 \pmod{100}$ holds.

You have 90 minutes to work on the problems and extra 25 minutes to scan and upload your solutions. Each problem is worth 10 points. To obtain a signature you have to achieve at least 24 points.

The details of the solutions must be explained, giving the result only is not worth any points. The problem marked with an * is supposed to be more difficult.