Introduction to the Theory of Computing I.
First midterm
October 30, 2020

1. Determine the numbers between 1 and 111 which, when multiplied by 1111, give the remainder 11 if we divide them by 2020.

2. What is the remainder if we divide 4^{74} by 70?

3. The plane S is given by the equation $-x+3y+6z=42$ and the line l is given by the equations $\frac{x-5}{2}=\frac{2-y}{3}=-\frac{z}{5}$. Determine the system of equations of the line which is perpendicular to S and passes through the intersection of S and l.

4. Let $V \subset \mathbb{R}^4$ be the set of vectors in $\mathbb{R}^4$ whose *first* two coordinates are equal. Similarly, let $W \subset \mathbb{R}^4$ be the set those vectors in $\mathbb{R}^4$ whose *last* two coordinates agree.

a) Decide if the set $V \cap W$ constitutes a subspace in $\mathbb{R}^4$ or not.

b) Decide if the set $V \cup W$ constitutes a subspace in $\mathbb{R}^4$ or not.

5. Determine those values of the real parameter p for which the following 3 vectors in $\mathbb{R}^4$ are linearly independent:

$$\underline{u}=\begin{pmatrix}1\\2\\3\\p\end{pmatrix},\ \underline{v}=\begin{pmatrix}1\\-1\\p\\8\end{pmatrix},\ \underline{w}=\begin{pmatrix}-2\\-1\\1\\p\end{pmatrix}.$$

6*. Assume that $f(n)=n^{n+1}$ and $g(n)=(n+2)^{n+3}$ for every positive integer $n \geq 1$. Show that the congruence $f(n)^{g(n)} \equiv 1 \pmod{g(n)}$ holds for infinitely many vaules of n.

Please work on stapled sheets only, and submit all of them at the end of the midterm. Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1, A2 or A3) on the first page.

You have 90 minutes to work on the problems, each of them is worth 10 points. To obtain a signature you have to achieve at least 18 points on each of the two midterm tests, and the total points from the two midterms should be at least 48.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used. The problem marked with an * is supposed to be more difficult.