1. Determine the numbers between 1 and 111 which, when multiplied by 1111, give the remainder 11 if we divide them by 2020.

2. What is the remainder if we divide $4^{74}$ by 70?

3. The plane $S$ is given by the equation $-x + 3y + 6z = 42$ and the line $l$ is given by the equations $\frac{x-5}{2} = \frac{2-y}{3} = -\frac{z}{5}$. Determine the system of equations of the line which is perpendicular to $S$ and passes through the intersection of $S$ and $l$.

4. Let $V \subset \mathbb{R}^4$ be the set of vectors in $\mathbb{R}^4$ whose first two coordinates are equal. Similarly, let $W \subset \mathbb{R}^4$ be the set those vectors in $\mathbb{R}^4$ whose last two coordinates agree.

   a) Decide if the set $V \cap W$ constitutes a subspace in $\mathbb{R}^4$ or not.

   b) Decide if the set $V \cup W$ constitutes a subspace in $\mathbb{R}^4$ or not.

5. Determine those values of the real parameter $p$ for which the following 3 vectors in $\mathbb{R}^4$ are linearly independent:

   $$ u = \begin{pmatrix} 1 \\ 2 \\ 3 \\ p \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ -1 \\ p \\ 8 \end{pmatrix}, \quad w = \begin{pmatrix} -2 \\ -1 \\ 1 \\ p \end{pmatrix}. $$

6*. Assume that $f(n) = n^{n+1}$ and $g(n) = (n + 2)^{n+3}$ for every positive integer $n \geq 1$. Show that the congruence $f(n)^{g(n)} \equiv 1 \pmod{g(n)}$ holds for infinitely many values of $n$.