Introduction to the Theory of Computing 1.
Repeated Second Midterm Test

2018. December 10.

1. Determine for which values of the parameter p the system of equations below is consistent. If it has solutions, then determine all of them.

$$\begin{aligned} x_1 + 4x_2 - 3x_3 &= 1 \\ 3x_1 + 17x_2 - 19x_3 &= 28 \\ 2x_1 + 11x_2 - 12x_3 &= p + 34 \\ 7x_1 + 26x_2 + p \cdot x_3 &= p + 15 \end{aligned}$$

2. Let A be the 4×4 matrix with 1's on and everywhere below its main diagonal, 2018 in the upper right corner, and 0's everywhere else. (I.e. it contains 10 1's and 5 0's.) Evaluate the determinant of A.

3. Compute the matrix A^{2018} for the matrix A below. (A^{2018} is the product with 2018 factors, each of whose is A.)

$$A = \begin{pmatrix} -5 & -4 & -8 \\ 1 & 0 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

4. Determine the missing entries of the matrices A and B (denoted by letters) if we know that $B = A^{-1}$ holds.

$$A = \begin{pmatrix} 5 & 2 & 0 \\ -1 & x & 1 \\ y & z & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -2 & p \\ q & 5 & r \\ s & -26 & 27 \end{pmatrix}$$

5. We know of the 4×6 matrix A that we can delete one appropriate row and column in such a way that the rank of the 3×5 matrix obtained is the same as the rank of A. Furthermore we know that if we delete two rows and two columns in any way, then the rank of the 2×4 matrix obtained is different from the rank of A. Determine the rank of A.

6. * For the $n \times n$ matrices A and $B \neq 0$ $A \cdot B = B$ holds. Prove that in this case there exists an $n \times n$ matrix C for which $C \neq 0$ and $A^T \cdot C = C$ holds.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (or other devices) are not allowed to use.