

# Introduction to the Theory of Computing 1.

## First Midterm Test

2018. October 18.

1. Determine the remainder we get if we divide  $4^{444}$  by 363.
2. The code written in C below calculates the square of the positive integer  $n$  (written in the decimal system). Suppose that the computer uses the “normal” basic operations (addition, subtraction, multiplication, division,...). Determine whether the algorithm is polynomial or not.

```
x = n; y = 0;
while (x > 0) {
    x = x-1;
    y = y+n;
}
printf(“Result: %d”, y);
```

3. \* Let  $n$  be a positive integer which is divisible by 8 but not divisible by 3. Prove that 3 is a witness of  $n$  (i.e. 3 proves the non-primality of  $n$  in the Fermat test).
4. Consider the line perpendicularly intersecting the plane  $2x - 4y + 7z = 2$  at the point  $(3, 1, 0)$ . Does this line intersect the line of system of equations  $\frac{x-1}{4} = \frac{2-y}{5} = \frac{z-3}{6}$ ?
5. Decide whether the vectors below form a generating system in  $\mathbf{R}^3$  or not.

$$\underline{a} = (3, 1, 0)^T, \underline{b} = (5, 2, 1)^T, \underline{c} = (3, 2, 3)^T$$

6. Decide whether the vectors in  $\mathbf{R}^4$  below are linearly independent or not.

$$\underline{u} = (2, 4, 3, 6)^T, \underline{v} = (3, 6, 2, 4)^T, \underline{w} = (1, 2, 0, 0)^T$$

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

Notes and calculators (and similar devices) cannot be used.

The question denoted by an \* is supposed to be more difficult.

Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 50-60 points: 5.