Introduction to Computer Science I.
Second Repeat of the Second Midterm Test
2016. December 16.

1. Determine all the values p for which the matrix below has an inverse and for all these values p determine the last entry in the last row of the inverse.
$$\begin{pmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 3 & 7 & p \end{pmatrix}$$

2. Is there a linear mapping $f : \mathbf{R}^2 \to \mathbf{R}^3$ for which $f((1,4)^T) = (0,1,2)^T$, $f((4,1)^T) = (5,4,3)^T$ and $f((2,2)^T) = (2,3,1)^T$ holds? If yes, then determine its matrix.

3. Let the matrix of the linear mapping $f : \mathbf{R}^4 \to \mathbf{R}^3$ be as below. Determine the dimension of the kernel and the image of f.
$$\begin{pmatrix} 2 & 3 & 1 & 3 \\ 1 & 2 & 0 & 4 \\ 0 & 1 & 0 & 5 \end{pmatrix}$$

4. Let a, b, c, d be fixed real numbers for which $ad - bc = 1$ holds. and let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation whose matrix in the basis $B = \{(a, b,)^T, (c, d)^T\}$ is $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$. Determine the matrix of f.

5. a) 2 is an eigenvalue of the matrix A. Is it true that 4 is an eigenvalue of the matrix A^2?
   b) 4 is an eigenvalue of the matrix B^2. Is it true that 2 or -2 is an eigenvalue of the matrix B?

6. Solve the linear congruence $34x \equiv 14 \pmod{59}$.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Calculators (or other devices) are not allowed to use.