Introduction to Computer Science I.
Second Midterm Test

2016. November 24.

1. Let A be a square matrix for which the matrix $A^2 - I$ has an inverse (where I denotes the identity matrix the same size as A). Is it true that the matrix $A + I$ has an inverse?

2. a) Let A be an arbitrary 6×6 matrix. Show that A can be written az the sum of 6 matrices of rank 1.
   b) Let B be a 6×6 matrix of rank 5. Show that B can be written az the sum of 5 matrices of rank 1.

3. Is there a linear transformation $f : \mathbf{R}^2 \to \mathbf{R}^2$ which maps the vectors $(1,0)^T$, $(1,1)^T$ and $(1,2)^T$ to the vector $(1,2)^T$? If yes, then determine its matrix.

4. Let $\underline{b}_1 = (2,1)^T$, $\underline{b}_2 = (3,1)^T$, $\underline{c}_1 = (1,1)^T$, $\underline{c}_2 = (1,0)^T$ and $f : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation and whose matrix in the basis $B = \{\underline{b}_1, \underline{b}_2\}$ is $[f]_B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$. Determine $[f]_C$, the matrix of f in the basis $C = \{\underline{c}_1, \underline{c}_2\}$.

5. Let A be the matrix below.
   a) Determine whether $\lambda = 2$ is an eigenvalue of A or not.
   b) Determine whether $(2,1,-1)^T$ is an eigenvector of A or not.
   c) Give an eigenvector of A whose first coordinate is 1 and the eigenvalue belonging to it.
$$A = \begin{pmatrix} 0 & 2 & 8 \\ 2 & 1 & 8 \\ 3 & 4 & 7 \end{pmatrix}$$

6. Determine all the solutions of the equation $17x + 23y = 601$ for which x and y are positive integers.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.