Introduction to Computer Science I.
Repeated First Midterm Test

2016. December 5.

1. The system of equations of the line e is $x = \frac{y}{3} = \frac{z}{5}$, and of the line f is $\frac{x}{-2} = \frac{3-y}{6} = \frac{2-z}{10}$. Decide whether e and f are parallel or not. If yes, then determine the equation of the plane containing both of them.

2. Do the vectors $(x, y)^T$ for which $x^2 = y^2$ holds form a subspace of $\mathbf{R}^2$?

3. Determine those values of the parameter p for which the vectors $(1, 3, 4, 2)^T$, $(10, 9, 10, p)^T$ and $(2, -1, -2, -1)^T$ are linearly independent.

4. A system of linear equations consisting of four equations has no such solutions for which the value of one of the variables is 0. Show that in this case there is a number $a \in \{1, 2, 3, 4, 5\}$ with the property that the system of linear equations has no such solutions in which the value of one of the variables is a either.

5. Determine all the nonzero elementary products and their signs arising from the evaluation of the determinant below when *using the original definition*.

$$\begin{vmatrix} 2 & 0 & 0 & 3 & 1 \\ 1 & 8 & 0 & 2 & 9 \\ 0 & 0 & 0 & 5 & 0 \\ 6 & 9 & 3 & 3 & 6 \\ 7 & 0 & 0 & 4 & 0 \end{vmatrix}$$

6. The determinant of a 4×4 matrix is 1. We call an entry of it *pleasant* if by changing that entry only the determinant of the matrix becomes 0. Show that the matrix has a pleasant entry in each row.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.