Introduction to Computer Science I.
Second Repeat of the Second Midterm Test

2015. December 18.

1. Suppose that the 3×3 matrix A has an inverse and each entry of both A and A^{-1} is an integer. What can be the value of the determinant of A?

2. What can be the rank of the matrix below (where p and q are real parameters)?
$$\begin{pmatrix} 1 & 2 & 3 & -1 \\ 3 & 6 & 9 & -3 \\ 1 & 0 & 1 & p \\ 3 & 4 & q & 2 \end{pmatrix}$$

3. For the linear mapping $f : \mathbf{R}^3 \to \mathbf{R}^4$ it holds that for each $\underline{x} \in \mathbf{R}^3$, it maps the vectors $\underline{x}$ and $(-\underline{x})$ to the same vector. Determine $[f]$, the matrix of f.

4. Let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation and $B = \{\underline{b}_1, \underline{b}_2\}$ and $C = \{\underline{c}_1, \underline{c}_2\}$ be two different bases in $\mathbf{R}^2$. Let the matrix of f in the basis B be the following matrix:
$$[f]_B = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$$
Determine $[f]_C$, the matrix of f in the basis C, if we know that $\underline{c}_1 = \underline{b}_1 + \underline{b}_2$ and $\underline{c}_2 = \underline{b}_1 - \underline{b}_2$.

5. The entries denoted by $\square$ of the matrix A below are unknown, but we know that 3 is an eigenvalue of A. Determine the other eigenvalue of A.
$$A = \begin{pmatrix} 4 & \square \\ \square & 6 \end{pmatrix}$$

6. In the 2×2015 matrix A the entry in the ith row and jth column is the remainder of $62 \cdot i \cdot j$ when divided by 2015, for each $1 \le i \le 2, 1 \le j \le 2015$. Does A have a column in which the first entry is exactly one less than the second entry? If yes, then for which columns does it hold?

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.