Introduction to Computer Science I.
Repeated Second Midterm Test

2015. December 7.

1. Determine the matrices A and B if the matrices A^{-1} and $A \cdot B$ are as below.

$$A^{-1} = \begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix} \qquad A \cdot B = \begin{pmatrix} -1 & 2 \\ 2 & -2 \end{pmatrix}$$

2. For the 6×10 matrix A $r(A) = 3$ holds. Is it always true that we can delete one row and one column of A, so that for the remaining 5×9 matrix B $r(B) = 2$ holds? (r denotes the rank of the matrix.)

3. The mapping $g : \mathbf{R}^3 \to \mathbf{R}^4$ maps every vector $(x_1, x_2, x_3) \in \mathbf{R}^3$ to the vector $(2x_1, 2x_2, 2x_3, 0) \in \mathbf{R}^4$. Let $B = \{\underline{b}_1, \underline{b}_2, \underline{b}_3\}$ be a basis in $\mathbf{R}^3$, and suppose that for the linear mapping $f : \mathbf{R}^3 \to \mathbf{R}^4$, $f(b_1) = g(b_1), f(b_2) = g(b_2)$ and $f(b_3) = g(b_3)$ holds. Determine $[f]$, the matrix of f.

4. Let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation, and let $B = \{\underline{b}_1, \underline{b}_2\}$ be a basis in $\mathbf{R}^2$. Let the matrix of f in the basis B be the matrix

$$[f]_B = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$

Determine whether the following statements are true or not:
a) $\underline{b}_1 + \underline{b}_s \in \mathrm{Ker} f$,
b) $\underline{b}_1 + \underline{b}_s \in \mathrm{Im} f$.

5. Determine the value of the parameter p if we know that the vector $\underline{v}$ below is an eigenvalue of the matrix A below. Determine all the eigenvalues and eigenvectors of the matrix A as well.

$$A = \begin{pmatrix} 6 & p \\ 9 & 6 \end{pmatrix}, \quad \underline{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

6. For some positive integer n, the last three digits of 6247 times n are 713. What can the last two digits of n be?

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.