Introduction to Computer Science I.
Repeated First Midterm Test
2015. December 7.

1. Determine the equation of the plane which passes through the point $Q(9,-2,5)$ and is perpendicular to the plane(s) $7x-2y+p\cdot z=4$ *for all values of the parameter p.*

2. Let $F=\{\underline{f}_1,\underline{f}_2,\ldots,\underline{f}_k\}$ be a linearly independent set, and $G=\{\underline{g}_1,\underline{g}_2,\ldots,\underline{g}_m\}$ a generating system in the subspace V of $\mathbf{R}^n$. Prove that we can add some (maybe 0) vectors of G to F to get a basis of V.

3. Let the set W consist of the vectors $\underline{v}\in\mathbf{R}^5$ for which it holds that the difference of any two coordinates of $\underline{v}$ is an integer. (E.g. the vector $(3.6,1.6,4.6,8.6,0.6)^T$ is like that.) Decide whether W forms a subspace in $\mathbf{R}^5$ or not. If yes, then determine the dimension of W.

4. Determine for which values of the parameters p and q the system of equations below is consistent. If it has solutions, then determine all of them.
$$\begin{aligned} x_1-3x_2-14x_3 &= -17\\ 2x_1-6x_2-28x_3+p\cdot x_4 &= q-34\\ 3x_1-7x_2-36x_3+4p\cdot x_4 &= 4q-37 \end{aligned}$$

5. Evaluate the determinant below *using the original definition.* (So don't use any properties of the determinant, or theorems about it during the solution, but determine the value using the definition only.)
$$\begin{vmatrix} 9 & 8 & 5 & 7 & 4\\ 0 & 2 & 0 & 0 & 6\\ 0 & 3 & 0 & 2 & 7\\ 0 & 1 & 0 & 0 & 3\\ 1 & 9 & 0 & 8 & 6 \end{vmatrix}$$

6. For the 5×3 matrix A it holds that the entry of the matrix $A\cdot A^T$ in the lower left corner is 2015. Determine the entry of $A\cdot A^T$ in the upper right corner.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.