

## List of Questions

1. Divisibility, prime numbers, fundamental theorem of arithmetic\*\*,  $d(n)$  function\*, greatest common divisor, least common multiple. Number of primes\*\*, gap between adjacent primes, prime number theorem (no proof).
2. Congruences, operations with congruences\*\*. Linear congruences, their solvability\*\* and methods for their solutions. Simultaneous congruence systems (example).
3. Euler's  $\varphi(n)$  function\*, reduced residue system. Euler-Fermat theorem\*\*, little Fermat theorem\*\*. Euclidean algorithm\*, its application for solving linear congruences (example).
4. Polynomial algorithms. Number theoretic algorithms: basic operations, exponentiation. Primality testing\*, public key cryptography, RSA-encoding\*.
5. Geometry of 3-space: equations of planes\*\*, lines\*\*; intersections. Dot product, cross product: definitions, properties, evaluation.
6. Definition of  $\mathbf{R}^n$  and subspaces of  $\mathbf{R}^n$ . Linear combination, spanned (generated) subspace, generating system, linear independence (2 definitions and their equivalence\*\*), exchange theorem\*\*, I-G inequality\*\*.
7. Basis, dimension\*\*. Standard basis, the dimension of  $\mathbf{R}^n$ \*. Coordinate vector in a basis, its uniqueness\*\*. Existence of a basis in a subspace of  $\mathbf{R}^n$ \*\*.
8. Systems of linear equations, Gaussian elimination. Row echelon form, reduced row echelon form. Conditions on consistency (solvability) and uniqueness\*\*.
9. Determinant: definition, basic properties\*\*, ways of evaluation, expansion theorem\*.
10. Matrices, operations on matrices, their properties. Product theorem for determinants (no proof). Connections between systems of linear equations and matrix equations\*\*.
11. Inverse of a matrix, necessary and sufficient condition for its existence\*\*, calculation of the inverse\*. Rank of a matrix: definitions, evaluation (no proof).
12. Linear maps: definition, basic properties, examples. Matrix of a linear map\*\*. Composition (product) of linear maps, its matrix\*. Inverse of a linear transformation\*\*.
13. Kernel and image of linear map\*\*, examples. Dimension theorem\*. Changing bases, the matrix of a linear transformation in a given basis\*\*.
14. Eigenvalues and eigenvectors of linear transformations and of matrices, their determination\*\*, characteristic polynomial, examples. Diagonalisation\*\*.

Theorems and statements with an \* were partially proved in the lecture.

Theorems and statements with a \*\* were completely proved in the lecture.