List of Questions

1. Divisibility, prime numbers, fundamental theorem of arithmetic**, $d(n)$ function*, greatest common divisor, least common multiple. Number of primes**, gap between adjacent primes, prime number theorem (no proof).

2. Congruences, operations with congruences**. Linear congruences, their solvability** and methods for their solutions. Simultaneous congruence systems (example).

3. Euler’s $\varphi(n)$ function*, reduced residue system. Euler-Fermat theorem**, little Fermat theorem**. Euclidean algorithm*, its application for solving linear congruences (example).


6. Definition of $\mathbb{R}^n$ and subspaces of $\mathbb{R}^n$. Linear combination, generated (spanned) subspace, generating system, linear independence (2 definitions and their equivalence**), exchange theorem**, I-G inequality**.

7. Basis, dimension**. Standard basis, the dimension of $\mathbb{R}^n$. Coordinate vector in a basis, its uniqueness**. Existence of a basis in a subspace of $\mathbb{R}^n$**.

8. Systems of linear equations, Gaussian elimination. Row echelon form. Conditions on consistency (solvability) and uniqueness**.


10. Matrices, operations on matrices, their properties. Product theorem for determinants (no proof). Connections between systems of linear equations and matrix equations**.


12. Linear maps: definition, basic properties, examples. Matrix of a linear map**. Composition (product) of linear maps, its matrix**. Inverse of a linear transformation**.

13. Kernel and image of linear map**, examples. Dimension theorem*. Changing bases, the matrix of a linear transformation in a given basis**.


Theorems and statements with an * were partially proved in the lecture.
Theorems and statements with a ** were completely proved in the lecture.