

List of Questions

1. Divisibility, prime numbers, fundamental theorem of algebra*, $d(n)$ function, greatest common divisor, least common multiple*.
2. Congruences, operations with congruences*. Linear congruences, their solvability* and methods for their solutions. Simultaneous congruence systems*.
3. Euler's $\varphi(n)$ function, reduced residue system. Euler-Fermat theorem*, little Fermat theorem*. Euclidean algorithm*, its application for solving linear congruences.
4. Polynomial algorithms. Number theoretic algorithms: basic operations, exponentiation. Primality testing*, public key cryptography, RSA-encoding*.
5. Geometry of 3-space: equations of planes*, lines*; intersections. Dot product, cross product: definitions, properties, evaluation.
6. Definition of \mathbf{R}^n and subspaces of \mathbf{R}^n . Linear combination, generated (spanned) subspace, generating system, linear independence (2 definitions and their equivalence*), exchange theorem (no proof), I-G inequality*.
7. Basis, dimension*. Standard basis, the dimension of \mathbf{R}^n . Coordinate vector, its uniqueness*. Existence of a basis in a subspace of \mathbf{R}^n .*
8. Systems of linear equations, Gaussian elimination. Row echelon form. Conditions on consistency (solvability) and uniqueness.
9. Determinant: definition, basic properties*, ways of evaluation, expansion theorem (partial proof).
10. Matrices, operations on matrices, their properties. Product theorem for determinants. Connections between systems of linear equations and matrix equations*.
11. Inverse of a matrix, necessary and sufficient condition for its existence*, calculation of the inverse. Rank of a matrix: definitions, evaluation.
12. Linear mappings: definition, basic properties, examples. Matrix of a linear mapping*. Composition (product) of linear mappings, its matrix*. Inverse of a linear transformation*.
13. Kernel and image of linear mappings*, examples. Dimension theorem*. Changing bases, the matrix of a linear transformation in a given basis*.
14. Eigenvalues and eigenvectors of matrices, characteristic polynomial*, examples. Diagonalisation*.

(Theorems and statements with a * were proved in the lecture.)