1. Not a subspace (not closed under multiplication by negative scalars).

2. a) Not a subspace (not closed under multiplication by large scalars),
   b) Subspace.

3. a) Not a subspace (not closed under multiplication by negative scalars),
   b) Subspace.

4. a) Not a subspace (not closed under addition).

5. Yes.

6. Not a subspace (not closed under multiplication by non-integer scalars).

7. No (not closed under addition).

8. No (not closed under addition).

9. No (not closed under addition).

10. a) Yes, \( \mathbf{a} = \mathbf{u} - \mathbf{v} + 2\mathbf{w} \).
    b) No. c) \( \{ \mathbf{z} \in \mathbb{R}^4 : 8x_1 - 4x_2 + 2x_3 - x_4 = 0 \} \).
    d) Same as in c).
    e) \( \mathbb{R}^4 \).

11. Those for which \( x_1 + x_3 = x_2 + x_4 \).

12. In the linear combination for \( \mathbf{v}_1 \) all the coefficients must be 0.

13. a) Plane, with equation \( 4x - 4y - z = 0 \),
    b) Line, with equation \( \frac{x}{2} = \frac{y}{2} = z \),
    c) Plane, with equation \( 5x - 7y + 2z = 0 \),
    d) Plane, with equation \( 5x - 7y + 2z = 0 \).

14. a) Line, with equation \( x = \frac{2y}{3} = \frac{4}{7} \).
    b) Plane, with equation \( 3x + y = 0 \).

15. Plane (since \( \mathbf{z} = -4\mathbf{a} + 3\mathbf{b} \)), with equation \( x - 3y + z = 0 \).

16. \( \{ \mathbf{z} \in \mathbb{R}^4 : x_1 + x_2 + 2x_3 - x_4 = 0 \} \).

17. Not a generating system.

18. Not a generating system.

19. Use the definition.

20. a) True.
    b) True (always, when \( \mathbf{u}, \mathbf{v} \) and \( \mathbf{w} \) are linear combinations of \( \mathbf{a}, \mathbf{b}, \mathbf{c} \)).

21. a) Find appropriate coefficients in the definition.
    b) Find appropriate coefficients in the definition.
    c) By contradiction.

22. Yes (by contradiction).

23. No, e.g. \( (1,0,0,0)^T, (0,1,0,0)^T, (\sqrt{2},0,0,0)^T \) is a counterexample.

24. Yes.

25. By contradiction.

26. Use the definition.

27. Linearly independent.

28. Linearly dependent.

29. Yes.