Introduction to the Theory of Computing 1.

Exercise-set 9.
Solutions

1. -3.

2. Evaluate the following determinant for all values of the parameter p.
a) $p-4$.
b) $10p^2+25p-320$.
c) $-4p+12$,
d) $-42p$.

3. a) 10,
b) 0.

4. a) 8,
b) -12.

5. $(c^2-d^2)^k$ (by induction).

6. $n \cdot \det(A)$.

7. No (if the cofactor belonging to this entry is 0).

8. a) No,
b) Yes.

9. Use the expansion theorem (for columns and rows).

10. $2x-5y+z-4=0$.

11. $7x+2y-z-4=0$

12. $9x+8y-7z-5=0$.

13. Parallel, $22x+25y-47z+1/3=0$.

14. Coplanar.

15. $p=1$ (the equation of the plane through $A,\ B,\ C$ is $5x-3y+z=4$).

16. a) -
b) $\begin{pmatrix} 3 & 5 & 7 \\ 2 & 1 & 0 \end{pmatrix}$,
c) -
d) $\begin{pmatrix} 5 & 9 & 13 \\ 4 & 3 & 2 \end{pmatrix}$,
e) $\begin{pmatrix} 14 & 6 \\ 6 & 3 \end{pmatrix}$.

17. a) all the entries of AB are 7,
b) 0.

18. a) yes,
b) no,
c) yes.

19. a) 0 matrix,
b) I,
c) $\begin{pmatrix} 1 & 2014 \\ 0 & 1 \end{pmatrix}$.

20. a) $A^2=I,\ B^2=B$ so $A^{2008}=I, B^{2008}=B$,
b) $\det(A)=1$ or -1, $\det(B)=1$ or 0.

21. a) $A^2=-I$ so $A^{2015}=-A$,
b) $\det(B^{2015})=\det(B)^{2015}$, so $\det(B^{2015})=7^{2015}$.

22. a) True: if $\det(A^k)=1$ then $\det(A)=\pm 1$.
b) False, e.g. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

23. There are no such matrices, because for those $\det(X) = \sqrt[2018]{2}$ would hold.

24. 2015 (the matrix $A \cdot A^T$ is symmetric).

25. $A_{ij}^2 = a_{i1}^2 + a_{i2}^2 + \ldots, a_{in}^2$, if it is 0, then $a_{i1} = a_{i2} = \ldots, a_{in} = 0$.

26. 2012(=sum of the squares of all the entries).

27. If $\det(A - I) = 0$ then $\det(A^2 - I) = \det((A - I)(A + I)) = 0$.

28. Yes, e.g. $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $B = I$.