Exercise-set 7.
Solutions

1. a) $x_1 = -42 + 11x_3,\ x_2 = 7 - 2x_3,\ x_3 \in \mathbf{R},\ x_4 = 4.$
   b) If $p = 21$, then same as in a), otherwise no solutions.

2. $x_1 = 2 + 2x_5 + x_3,\ x_2 = 2 - x_5 - 2x_3,\ x_3 \in \mathbf{R},\ x_4 = 2,\ x_5 \in \mathbf{R}.$

3. If $c = 7$, then $x_1 = 20 + 11x_3 + 3x_5,\ x_2 = -8 - 3x_3 - 2x_5,\ x_3 \in \mathbf{R},\ x_4 = 3 + 2x_5,\ x_5 \in \mathbf{R}.$
   If $c \neq 7$, then $x_1 = 17 + 11x_3,\ x_2 = -6 - 3x_3,\ x_3 \in \mathbf{R},\ x_4 = 1,\ x_5 = -1.$

4. If $p = 0,\ q \neq 0$, then no solutions.
   If $p = 0,\ q = 0$, then $x_1 = 4 + 5x_3,\ x_2 = 7 - 3x_3,\ x_3 \in \mathbf{R},\ x_4 \in \mathbf{R}.$
   If $p \neq 0$, then $x_1 = 4 + 5x_3,\ x_2 = 7 - 3x_3,\ x_3 \in \mathbf{R},\ x_4 = q/p.$

5. a) If $p = -1$, then no solutions.
   If $p = 0$, then $x_1 = -3 - 6x_4,\ x_2 = -1 - 2x_4,\ x_3 = 1,\ x_4 \in \mathbf{R}.$
   If $p \neq 0, -1$, then $x_1 = -3 + 6/(p+1),\ x_2 = -1 + 2/(p+1),\ x_3 = 1,\ x_4 = -1/(p+1).$

   b) If $p = 13$, then no solutions.
   If $p \neq 13$, then $x_1 = 1 - 1/(p-13) + 3x_2,\ x_3 = 1 - 2/(p-13),\ x_4 = 1/(p-13).$

   c) If $p \neq 16$, then no solutions.
   If $p = 16$, then $x_1 = -32 + 16x_3,\ x_2 = 6 - 3x_3,\ x_3 \in \mathbf{R},\ x_4 = 1.$

6. a) $\frac{x-18}{-5} = \frac{y+12}{4} = z.$
   b) no intersection.

7. a) lin. dependent, not a basis.
   b) lin. independent, basis. $[\underline{v}]_B = (7, -7, -1, 4)^T$

8. Only for $p = 6$.

9. Only for $p = 0$, then $[\underline{v}]_B = (-2, -1, 1, 3)^T$.

10. a) Yes, e.g. $x_1 = 0,\ x_1 = 0.$
    b) No (no free parameters).

11. At least 5.

12. There are no free parameters. If there are no solutions, it holds. If there is a unique solution, then at most 4 values can be used in it.

13. By contradiction: 3 nonzero rows $\Longrightarrow$ 2 free parameters, but then their values can be equal. So TRUE.

14. If $p \neq 2$, then $x_1 = 4,\ x_2 = -1,\ x_3 = 1.$
    If $p = 2$, then $x_1 = 2 + 2x_3,\ x_2 = 1 - 2x_3,\ x_3 \in \mathbf{R}.$

15. If $p \neq 0$, then $x_1 = 2 + \frac{4p+3}{p}x_4,\ x_2 = 1 + \frac{1}{p}x_4,\ x_3 = -\frac{p+1}{p}x_4,\ x_4 \in \mathbf{R}.$
    If $p = 0$, then $x_4 = 0,\ x_3 \in \mathbf{R}, x_1 = 2 - 3x_3,\ x_2 = 1 - x_3.$

16. If $p = 0$, then $x_1 = -1 - 19x_3,\ x_2 = 1 + 6x_3,\ x_3 \in \mathbf{R}.$
    If $p \neq 0$, then $x_1 = -22/3,\ x_2 = -1,\ x_3 = -1/3.$