Exercise-set 5.
Solutions

1. Not a subspace (not closed under multiplication by negative scalars).
2. a) Not a subspace (not closed under multiplication by large scalars),
   b) Subspace.
3. a) Not a subspace (not closed under multiplication by negative scalars),
   b) Subspace.
4. $W_1 \cup W_2$ is not a subspace (not closed under addition), but $W_1 \cap W_2$ is.
5. a) Yes,
   b) No (not closed under addition).
6. Yes.
7. Not a subspace (not closed under multiplication by non-integer scalars).
8. No (not closed under addition).
9. No (not closed under addition).
10. a) Yes, $\underline{a} = \underline{u} - \underline{v} + 2\underline{w}$.
    b) No. c) $\{\underline{x} \in \mathbf{R}^4 : \ 8x_1 - 4x_2 + 2x_3 - x_4 = 0\}$.
    d) Same as in c).
    e) $\mathbf{R}^4$.
11. Those for which $x_1 + x_3 = x_2 + x_4$.
12. In the linear combination for $\underline{v}_1$ all the coefficients must be 0.
13. a) Plane, with equation $4x - 4y - z = 0$,
    b) Line, with equation $\frac{x}{2} = \frac{y}{-5} = z$,
    c) Plane, with equation $5x - 7y + 2z = 0$,
    d) Plane, with equation $5x - 7y + 2z = 0$.
14. Plane, with equation $16x - 5y - 7z = 0$.
15. a) Line, with equation $x = \frac{y}{-3} = \frac{z}{4}$.
    b) Plane, with equation $3x + y = 0$.
16. Plane (since $\underline{c} = -4\underline{a} + 3\underline{b}$), with equation $x - 3y + z = 0$.
17. $\{\underline{x} \in \mathbf{R}^4 : \ x_1 + x_2 + 2x_3 - x_4 = 0\}$.
18. Use the definition.
19. Yes.
20. Yes.
21. a) True.
    b) True (always, when $\underline{u}, \underline{v}$ and $\underline{w}$ are linear combinations of $\underline{a}, \underline{b}, \underline{c}$.
22. Find approriate coefficients in the definition.
23. Find approriate coefficients in the definition.
24. By contradiction.
25. Check the cases $\lambda_1 = 0$ and $\lambda_1 \neq 0$
26. Yes (by contradiction).
27. No, e.g. $(1,0,0,0)^T, (0,1,0,0)^T, (\sqrt{2},0,0,0)^T$ is a counterexample.
28. By contradiction.
29. Use the definition.
30. Linearly independent.