Introduction to the Theory of Computing 1.

Exercise-set 4.
Solutions

1. a) If $g.c.d.(a, 651) = 1$, then $a^2 \equiv 1 \pmod 3$, $a^{10} \equiv 1 \pmod{11}$ and $a^{16} \equiv 1 \pmod{17}$, so $a^{560} \equiv 1 \pmod{561}$.
   b) $g.c.d.(a, 1105) = 1$, then $a^4 \equiv 1 \pmod 5$, $a^{12} \equiv 1 \pmod{13}$ and $a^{16} \equiv 1 \pmod{17}$, so $a^{1104} \equiv 1 \pmod{1105}$.
2. The decoding function is $x \mapsto x^3 \pmod{85}$, the original message is SECRET.
3. Substitute, $t = -3$, $P = (-2, 7, 7)$.
4. $4x - y - 2z = 11$.
5. $x = 2 + t, y = -1 + 3t, z = -5t$.
6. $\underline{n} = (4, 0, -1) = \underline{v}$, $x = 2 + 4t, y = -5, z = -2 - t$.
7. $\underline{v} = (2, -2, 3) = \underline{n}$, $2x - 2y + 3z = -9$.
8. $5x - 4y + 3z = 0$, yes.
9. $\underline{v} = (4, -4, 2)$, $x = 2 + 2t, y = 7 - 2t, z = 3 + t$. Yes, $t = 5$.
10. a) Yes, both planes contain P,
    b) Yes, $\underline{n}_1 \cdot \underline{n}_2 = 0$.
11. $x = 6 + t, y = 3 + 4t, z = -1 - 4t$, $(4 + t) + 4(9 + 4t) - 3(-4 - 3t) = 0$, $t = -2$, $P = (4, 5, 7)$.
12. Same question as previous, $P = (2, -1, 3)$.
13. Yes, $P = (0, 0, 7)$.
14. $M = (2, 1, 3)$, $\underline{n} = (1, 0, 2)$. Doesn't intersect the y axis.
15. $9x + 8y - 7z = 5$.
16. a) $p = 3, q \neq 4$ (no solutions),
    b) $p \neq 3$ (exactly one solution),
    c) $p = 3, q = 4$ (infinitely many solutions).
17. $\underline{v}_1 = \underline{v}_2 = (2, 2, 2)$, they are parallel. $22x + 25y - 47z = -\frac{1}{3}$.
18. They have a common point, namely $(1, 1, 1)$.
19. They are parallel; $-9x - 2y + 3z = 0$.
20. $p = 1$, $q = 2$.
21. $2x - y + 3z = 23$, no.
22. $-5x + 4y + 4z = 1$, yes, $(0, 1/4, 0)$.
23. $x = 413/71 - 3t, y = 431/71 + 2t, z = 40/71 - 3t$.