Exercise-set 11.
Solutions

1. The dimension of the subspace = rank of the matrix with these columns = 2.

2. a) $B = [A'^{-1}|0]$ works if the first 2×2 minor of A, A' has nonzero determinant.
   b) Use 5.a)

3. a) True ($\det(A) \neq 0$),
   b) False ($\det(B) = 0$),
   c) Might or might not be true (examples).

4. a) Columnspace of $A \cdot B \subset$ columnspace of A.
   b) Columnspace of $A + B \subseteq$ $\langle$columnspace of A, columnspace of $B\rangle$.

5. row rank=column rank.

6. No (determinant rank).

7. a) $r = 1$: No (counterexample), $r = 3$: Yes.
   b) Changing 1 element can create at most 1 new linerly independent row.
   c) Yes.

8. a) Use determinant rank.
   b) No (counterexample).

9. a) A has r linearly independent rows: $a_1, a_2, \ldots, a_r$, the others are linear combinations of these. The kth row of the matrix A_i should be that multiple of a_i which appears in the kth row of A.
   b) Similar to a).

10. a) True,
    b) True.

11. a) $[f] = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$, $\mathrm{Ker} f = \{(x, y, z)^T \in \mathbf{R}^3 : x - y + z = 0\}$, $\mathrm{Im} f = \{(x, y)^T \in \mathbf{R}^2 : x = y\}$,
    b) $[f] = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\mathrm{Ker} f = \{\underline{0}\}$, $\mathrm{Im} f = \mathbf{R}^2$, c) Not a linear mapping,
    d) $[f] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $\mathrm{Ker} f = \{\underline{0}\}$, $\mathrm{Im} f = \{(x, y, z, w)^T \in \mathbf{R}^4 : x + y + z = w\}$, e) Not a linear mapping,
    f) $[f] = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, $\mathrm{Ker} f = \{(x, y)^T \in \mathbf{R}^2 : x + y = 0\}$, $\mathrm{Im} f = x$ axis, g)

12. No.

13. a) $[f] = \begin{pmatrix} -5 & 5 \\ -6 & 6 \end{pmatrix}$,
    b) $(5, 6)^T$.

14. $[f] = 0$.

15. (a) 0,
    (b) 1,
    (c) infinitely many.

16. a) False,
    b) True,
    c) False,
    d) True.