

Exercise-set 10.
Solutions

1. $\begin{pmatrix} 4 & 0 \\ -1 & 0 \\ 2 & 0 \end{pmatrix}$

2. $B = \begin{pmatrix} 1-2x & 2-2y \\ x & y \end{pmatrix}$ for $x, y \in \mathbf{R}$.

3. a) $\begin{pmatrix} -6 & -1 & 3 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 3 & -2 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 4 & -3 & -1 & 1 \\ -1 & 1 & 0 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 2 & 0 & -1 & 0 \\ 1 & 1 & -2 & 1 \\ -2 & -1 & 2 & 0 \\ 1 & 1 & -1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} n & -1 & -1 & \dots & -1 \\ -1 & 1 & 0 & \dots & 0 \\ -1 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 0 & 0 & \dots & 1 \end{pmatrix}$ e) $\begin{pmatrix} 2 & -1 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 \\ 0 & 0 & -1 & 2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}$

4. a) $p \neq 0$; $A^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ -2 & 1 & 0 \\ 5/p & -4/p & 1/p \end{pmatrix}$.

b) $p \neq 11$; $1/(p-11)$.

c) $p \neq 0$; $7/2$.

5. Multiply by A^{-1} from the left.

6. A cannot be invertible.

7. Yes, $X = A^{-1}B$.

8. a) Yes.

b) Yes.

9. $\det(A) = \pm 1$.

10. $A = (A^{-1})^{-1}$, $B = A^{-1} \cdot (A \cdot B)$.

11. $(X \cdot Y)^{-1} = A$

12. Yes (e.g. the first 2 columns of B are 0, the last 3 is A^{-1}).

13. YES. ($A^2 - I = (A + I)(A - I)$.)

14. a) $r = 2$,

b) $r = 2$,

c) $r = 3$,

d) $r = 1$.

15. a) If $p = 1$ or $p = 1/2$ then $r = 2$, otherwise $r = 3$.

b) If $p = 4$, then $r = 3$, otherwise $r = 4$.

c) If $p = 1$ then $r = 1$, if $p = 0$ then $r = 2$, otherwise $r = 3$.

16. No.

17. $r(A) = 1$ is not possible, $r(A) = 2$ if $r = 2$, $s = 1$.

18. a) If $pq = 1$, then $r = 2$, otherwise $r = 3$.

b) $r = 2$ for all values of x .

c) If $x = 6$, then $r = 2$, otherwise $r = 3$.

19. At most 2 leading 1's.