Exercise-set 5.

1. Determine the common point of the line \( x = 1 + t, y = -2 - 3t, z = 7 \) and the plane \( x + 2y - z = 5 \).

2. Determine the equation of the plane through the point \( P(3, -1, 1) \) parallel to the plane \( 4x - y - 2z - 6 = 0 \).

3. Determine the system of equations of the line through the point \( P(2, -1, 0) \) parallel to the line \( x + 1 = \frac{y - 2}{3} = \frac{1 - z}{5} \).

4. Determine the system of equations of the line through the point \( P(2, -5, -2) \) perpendicular to the plane \( z = 4x + 7 \).

5. (MT'+’07) Determine the equation of the plane, which is perpendicular to the line of system of equations \( \frac{2 - x}{5} = \frac{y - 10}{2} = \frac{z + 8}{3} \) and goes through the point \( P(1, 4, -1) \).

6. (MT’10) Consider the plane which is parallel to the plane of equation \( 5x - 4y + 3z = 9 \) and contains the point \( P(1, 5, 5) \). Does this plane pass through the origin?

7. (MT’06) Determine whether the line through the points \( P(2, 7, 3) \) and \( Q(6, 3, 5) \) contains the point \( R(12, -3, 8) \) or not.

8. (MT++’11) Given the planes \( S_1 \) of equation \( 2x + y - 3z = 2 \) and \( S_2 \) of equation \( x + 7y + 3z = 21 \) determine whether
   a) their line of intersection contains the point \( P(5, 1, 3) \) or not;
   b) \( S_1 \) and \( S_2 \) are perpendicular to each other or not.

9. (MT’15) Determine the point(s) \( P \) on the line \( l \) with system of equations \( x - 6 = \frac{y - 3}{4} = \frac{z - 2}{3} \) for which the line connecting \( P \) with \( Q(2, -6, 5) \) is perpendicular to \( l \).

10. (MT++’10) Determine the equation of the line passing through the point \( P(12, 1, 7) \) and perpendicularly intersecting the line of system of equations \( x - 3 = \frac{y - 2}{5} = \frac{z - 1}{4} \).

11. (MT’06) Determine whether the line through the points \( P(1, 4, 4) \) and \( Q(3, 12, -2) \) intersects one of the coordinate axes or not. If yes, determine the point(s) of intersection.

12. (MT’13) Determine the equation of the perpendicular bisector plane of the line segment connecting the points \( P = (1, 1, 1) \) and \( Q = (3, 1, 5) \) in 3-space (i.e. the equation of the plane which is perpendicular to the line segment \( PQ \) and goes through its midpoint). Where does this plane intersect the \( y \) axis?

13. For which values of the parameters \( p, q \) will the planes \( 2x + 3y - z = 6, x - 3y + 2z = 5 \) and \( 4x - 3y + pz = q \) a) have no common point,
    b) have exactly one point in common,
    c) have a common line.

14. (MT++’14) Determine whether the line given by the system of equations \( \frac{5x + 3}{10} = \frac{x + 1}{2}, \frac{3 - 2z}{2} \) is parallel to the line of intersection of the planes \( 6x + y + 7z = 91 \) and \( 5x + 2y = 79 \) or not.

15. (MT’16) We know that the line \( e \) perpendicularly intersects the plane of equation \( x + 2y + 3z = 6 \) at the point \( (1, 1, 1) \); moreover, that the line \( f \) contains both the points \( (5, 2, -1) \) and \( (13, 4, -5) \). Decide whether \( e \) and \( f \) have a common point or not.

16. (MT++’16) The system of equations of the line \( e \) is \( x = t + 1, y = 2t + 1, z = 2t + 1 \), and the equation of the plane \( S \) is \( 4x - 3y + pz = q \). Determine all the values \( p \) and \( q \) for which the line \( e \) is in the plane \( S \).
17. (MT’17) Consider the plane which perpendicularly intersects the line connecting \( P(3, -2, 5) \) and \( Q(7, -4, 11) \) in \( P \). Does this plane contain the point \( R(-4, 1, 3) \)?

18. (MT’18) Consider the line perpendicularly intersecting the plane \( 2x - 4y + 7z = 2 \) at the point \( (3, 1, 0) \). Does this line intersect the line of system of equations \( \frac{x - 1}{2} = \frac{y - 1}{3} = \frac{z - 3}{4} \)?

19. (MT+’18) * Consider the triangle with vertices \( A(1, 5, 2), B(2, 7, 4) \) and \( C(2, 9, 10) \). Determine the (inner) bisector of the angle at vertex \( A \). (The bisector of the angle at \( A \) is the line through \( A \) in the plane of the triangle for which the angles enclosed by it and the sides \( AB \) and \( AC \) respectively are equal.)

20. (MT++’18) The ray of light starting from the light source at the point \( P(3, 17, 27) \) falls perpendicularly on the plane mirror of equation \( 3x - y - 2z = 8 \). Determine the point of incidence. (We can suppose that the light is spread in a straight line.)