Introduction to the Theory of Computing 1.

Exercise-set 4.

1. a) Prove that $561 = 3 \cdot 11 \cdot 17$ is a Carmichael number.
   b) Prove that $1105 = 5 \cdot 13 \cdot 17$ is a Carmichael number.

2. We substitute the 26 letters of the alphabet by the numbers $0, 1, \ldots, 25$ (so $A = 0,\ B = 1,\ C = 2, \ldots, Z = 25$). The public key encoding function is $x \mapsto x^{43} \pmod{85}$. (With this we can encode the numbers $0, 1, \ldots, 84$ but only the first 26 numbers have meaning.) What is the original message if the one encoded by this function is 52 64 8 68 64 59?

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3. Determine the common point of the line $x = 1 + t, y = -2 - 3t, z = 7$ and the plane $x + 2y - z = 5$.

4. Determine the equation of the plane through the point $P(3, -1, 1)$ parallel to the plane $4x - y - 2z - 6 = 0$.

5. Determine the system of equations of the line through the point $P(2, -1, 0)$ parallel to the line $x + 1 = \frac{y-2}{3} = \frac{4-z}{5}$.

6. Determine the system of equations of the line through the point $P(2, -5, -2)$ perpendicular to the plane $z = 4x + 7$.

7. (MT+'07)Determine the equation of the plane, which is perpendicular to the line of system of equations $\frac{x-5}{2} = \frac{y-10}{-2} = \frac{z+8}{3}$ and goes through the point $P(1, 4, -1)$.

8. (MT'10) Consider the plane which is parallel to the plane of equation $5x - 4y + 3z = 9$ and contains the point $P(1, 5, 5)$. Does this plane pass through the origin?

9. (MT'06) Determine whether the line through the points $P(2, 7, 3)$ and $Q(6, 3, 5)$ contains the point $R(12, -3, 8)$ or not.

10. (MT++'11) Given the planes S_1 of equation $2x + y - 3z = 2$ and S_2 of equation $x + 7y + 3z = 21$ determine whether
    a) their line of intersection contains the point $P(5, 1, 3)$ or not;
    b) S_1 and S_2 are perpendicular to each other or not.

11. (MT'15) Determine the point(s) P on the line l with system of equations $x - 6 = \frac{y-3}{4} = \frac{1-z}{3}$ for which the line connecting P with $Q(2, -6, 5)$ is perpendicular to l.

12. (MT+'10) Determine the equation of the line passing through the point $P(12, 1, 7)$ and perpendicularly intersecting the line of system of equations $x - 3 = \frac{y-2}{3} = \frac{-z-1}{4}$.

13. (MT+'06) Determine whether the line through the points $P(1, 4, 4)$ and $Q(3, 12, -2)$ intersects one of the coordinate axes or not. If yes, determine the point(s) of intersection.

14. (MT'13) Determine the equation of the perpendicular bisector plane of the line segment connecting the points $P = (1, 1, 1)$ and $Q = (3, 1, 5)$ in 3-space (i.e. the equation of the plane which is perpendicular to the line segment $\overline{PQ}$ and goes through its midpoint). Where does this plane intersect the y axis?

15. (MT'12) Determine the equation of the plane which passes through the points $P(1, 3, 4)$ and $Q(3, 6, 10)$ and is parallel to the line given by the system of equations $\frac{x-9}{3} = y + 4 = \frac{z}{5}$.

16. For which values of the parameters $p,\ q$ will the planes $2x + 3y - z = 6,\ x - 3y + 2z = 5$ and $4x - 3y + pz = q$
    a) have no common point,
    b) have exactly one point in common,
    c) have a common line.

17. (MT+'14) Determine whether the lines e and f given by the systems of equations below are parallel or not. If yes, then determine the equation of the plane S containing them.

$$e: \ \frac{2x-3}{4} = \frac{3y+4}{6} = \frac{z}{2} \qquad f: \ \frac{x+1}{2} = \frac{y-4}{2} = \frac{3z-5}{6}$$

18. (MT'16) We know that the line e perpendicularly intersects the plane of equation $x + 2y + 3y = 6$ at the point $(1, 1, 1)$; moreover, that the line f contains both the points $(5, 2, -1)$ and $(13, 4, -5)$. Decide whether e and f have a common point or not.

19. (MT+'16) The system of equations of the line e is $x = \frac{y}{3} = \frac{z}{5}$, and of the line f is $\frac{x}{-2} = \frac{3-y}{6} = \frac{2-z}{10}$. Decide whether e and f are parallel or not. If yes, then determine the equation of the plane containing both of them.

20. (MT++'16) The system of equations of the line e is $x = t + 1,\ y = 2t + 1,\ z = 2t + 1$, and the equation of the plane S is $4x - 3y + pz = q$. Determine all the values p and q for which the line e is in the plane S.

21. (MT'17) Consider the plane which perpendicularly intersects the line connecting $P(3, -2, 5)$ and $Q(7, -4, 11)$ in P. Does this plane contain the point $R(-4, 1, 3)$?

22. (MT+'17) Does the plane through the points $A(-1, -2, 1),\ B(3, 1, 3)$ and $C(7, 6, 3)$ contain a point which is on the y axis? If yes, then which is it?

23. (MT++'17) The system of equations of the line e is $\frac{x+3}{5} = \frac{y+1}{9} = z$, and of the line f is $\frac{x}{4} = \frac{y+8}{6}, z = 7$. Determine the system of equations of the *normal transversal* of e and f, that is, of the line n which intersects both e and f perpendicularly.