Introduction to the Theory of Computing 1.

Exercise-set 3.

1. The code written in C below calculates the sum of the positive integers a and b (written in the decimal system). Suppose that the computer uses the "normal" basic operations (addition, subtraction, multiplication, division,...). Determine whether the algorithm is polynomial or not.

```
while (b > 0) {
   a = a+1;
   b = b-1;
 printf(‘‘Sum: %d’’, a);
```

2. The code written in C below calculates the sum of the positive integers a and b (written in the decimal system). Suppose that the computer uses the "normal" basic operations (addition, subtraction, multiplication, division,...). Determine whether the algorithm is polynomial or not. (`ceil(b/2.0)` gives the upper integer part, and `floor(b/2.0)` the lower integer part of $\frac{b}{2}$ back.)

```
while (b > 0) {
   a = a + ceil(b/2.0);
   b = floor(b/2.0);
}
printf(‘‘Sum: %d’’, a);
```

3. The code written in C below calculates the largest divisor of n not greater than a (for the integers $0 < a < n$ written in the decimal system). Suppose that the computer uses the "normal" basic operations (addition, subtraction, multiplication, division,...). Determine whether the algorithm is polynomial or not.

```
while (n % a != 0) {
   a = a-1;
}
printf(‘‘Result: %d’’, a);
```

4. The code written in C below calculates $\lfloor\sqrt{n}\rfloor$ (for the positive integer n written in the decimal system). Suppose that the computer uses the "normal" basic operations (addition, subtraction, multiplication, division,...). Determine whether the algorithm is polynomial or not.

```
X = 0; y = 0;
while (y <= n) {
   x = x+1;
   y = x * x;
}
printf(‘‘Result: %d’’, x-1);
```

5. The code written in C below calculates $\lfloor\log_2 n\rfloor$ (for the positive integer n written in the decimal system). Suppose that the computer uses the "normal" basic operations (addition, subtraction, multiplication,...). Determine whether the algorithm is polynomial or not.

```
X = 0; y = 1;
while (y <= n) {
   x = x+1;
   y = 2 * y;
}
printf(‘‘Result: %d’’, x-1);
```

6. Calculate the value of the following expressions (you can use a calculator for it, exceptionally):
   a) the remainder of 3^{45} when divided by 79;
   b) the remainder of 5^{300} when divided by 623;
   c) the g.c.d. of 673 and 101;
   d) the g.c.d. of 346 and 158;
   e) the integer solutions of the congruence $101x \equiv 3 \pmod{673}$;
   f) the integer solutions of the congruence $119x \equiv 2 \pmod{514}$;
   g) the integer solutions of the congruence $155x \equiv 7 \pmod{352}$.

7. (MT'17+) Use the algorithm we learnt to determine the remainder we get if we divide 5^{85} by 155.

8. (MT'17++) Let $n = 123456$. Use the algorithm we learnt to determine the g.c.d. of $12n + 6$ and $9n + 4$.