Introduction to the Theory of Computing 1.

Exercise-set 2.

1. Solve the following linear congruences:
a) $3x \equiv 2 \pmod 5$;
b) $10x \equiv 12 \pmod{50}$;
c) $26x \equiv 6 \pmod{110}$;
d) $32x \equiv 12 \pmod{82}$;
e) $5x \equiv 1 \pmod{28}$;
f) $9x \equiv 1 \pmod{88}$;
g) (MT'04) $170x \equiv 78 \pmod{2006}$.

2. a) What can the remainder of an integer be when divided by 34 if 26 times that integer gives a remainder of 16 when divided by 34?
b) We know that for some integer n, when we divide $41n - 27$ and $n + 1$ by 62, we get the same remainder. What can this common remainder be?
c) Determine the two-digit integer n, if we know that the last two digits of $78n + 1$ and $39n + 2$ are the same.

3. (MT++'09) Determine all the three-digit integers which are divisible by 17 and give a remainder of 2 when divided by 50.

4. (MT+'11) What can the remainder of an integer n be when divided by 202 if $53n - 1$ is divisible by 202?

5. (MT'13) The remainder of an integer when divided by 222 is 4 less than the remainder of 60 times that integer when divided by 222. What can the remainder of this integer be when divided by 222?

6. (MT+'13) We know that for some integer n, when we divide $37n + 9$ and $n + 10$ by 235, we get the same remainder. What can this common remainder be?

7. (MT'13) The remainder of the integer n when divided by 82 is 3. What can the remainder of n be when divided by 182?

8. (MT'14) For the integer n, the last two digits of $43n - 1$ and $2n + 2$ are the same. What are these two digits?

9. (MT+'14) The remainder of an integer when divided by 109 is 5 less than the remainder of 18 times that integer when divided by 109. What can the remainder of this integer be when divided by 109?

10. (MT'15) What can the remainder of an integer n when divided by 166 be if $71n + 21$ and $33 - 29n$ give the same remainder when divided by 166?

11. (MT+'15) For some positive integer n, the last three digits of 6247 times n are 713. What can the last two digits of n be?

12. (MT'17) The remainder of 107 times an integer when divided by 532 is 102 more than the remainder of the integer itself when divided by 532. What can the remainder of this integer be when divided by 532?

13. (MT'17++) How many integers x are there between 1 and 2017 for which it holds that $92x - 1$ and x give the same remainder when divided by 399?

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14. For which integers does it hold that they give a remainder of 2 when divided by 7 and a remainder of 3 when divided by 9?

15. (MT'12) How many positive integers are there which are less than 2012 that give a remainder of 10 when divided by 19 and give a remainder of 15 when divided by 37?

16. (a) A millipede wants to count its legs. It knows that every millipede has at most 344 legs. If it counts its legs by 13's then 3 are left out, and if by 17's then 10 are left out. How many legs does the millipede have?
(b) Another millipede wants to use this method as well. If it counts its legs by 16's then 5 are left out, and if by 20's then 15. Show that it made a mistake.
(c) The king of millipedes learns about the method also. If it counts its legs by 6's then 5 are left out, by 7's then 6 are left out and if by 8's then 7. How many legs does it have?

17. The last digit of an integer in the numerical system of base 20 is '11'. What can its last digit be in the numerical system of base
(a) 9,
(b) 8?

18. (MT'17) The last digit of an integer in the numerical system of base 16 is '13'. What can its last digit be in the numerical system of base 12?

19. (MT'17+) Determine all the four-digit integers which give a remainder of 3 when divided by 51, furthermore if we multiply them by 17, then the last two digits of the product are 15.

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20. (MT'09) How many positive integers a are there which are not greater than 600 and for which the linear congruence $a \cdot x \equiv 1 \pmod{600}$ has a solution?

21. (MT+'09) How many positive integers a are there which are not greater than 540 and for which the linear congruence $a \cdot x \equiv 6 \pmod{540}$ has exactly 3 solutions?

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22. Determine the remainder we get if we divide
a) 2^{100} by 45,
b) (MT'14) $46^{47^{48}}$ by 25.

23. Determine the last two digits of
a) 303^{404},
b) (MT+'10) $33^{21^{34}}$,
c) 159^{161},
d) $49^{49^{50}}$,
e) (MT++'03) $17^{17^{17}} - 17^{17} + 17$.

24. (MT'07) Let $n = 200705111601$. Determine the last three digits of n^n.

25. (MT+'13) Determine the last two digits of the number $42^{41^{40}}$ in the numerical system of base 11.

26. (MT'05) Let A be the arithmetic progression whose first term is 32, and whose difference is 51. (So the first few terms of A are $32, 83, 134, \ldots$) Determine the remainder we get if we divide the product of the first 32 terms of A by 51.

27. (MT'11) What is the remainder if we divide $100^{3^{2011}}$ by 3^{2011}?

28. (MT'14) The integer n written in the binary system is 110100101101100011011. Determine the last 4 digits of n^n in the binary system.

29. (MT'17) Determine the remainder we get if we divide 799^{801} by 264.

30. (MT'17+) Determine the remainder we get if we divide $73^{37} + 37^{73}$ by 108.

31. (MT'17++) Let p be a positive prime number different from 3 and a be an integer not divisible neither by 3 nor by p. Show that in this case

$$a^{6p-6} \equiv 1 \pmod{9p}$$