Introduction to the Theory of Computinge 1.

Exercise-set 13.

1. MT+'12 Determine all the eigenvalues and eigenvectors of the following matrix:

$$\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

2. Let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation which maps an arbitrary vector $(x, y) \in \mathbf{R}^2$ into the vector $(2x + y, 3x + 4y)$.
a) Determine all the eigenvalues and eigenvectors of $[f]$, the matrix of f.
b) Determine a basis B consisting of the eigenvectors of f, and give $[f]_B$, the matrix of f in the basis B.

3. Determine the eigenvalues and eigenvectors of the following matrix:

$$\begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 4 & 3 \end{pmatrix}$$

4. MT+'10 Let A be the 5×5 matrix with 2's on its main diagonal, and 1's everywhere else.
a) Determine one eigenvalue of A.
b) Determine one eigenvector of A.

5. MT'14 Is 3 an eigenvalue of the matrix A below? If yes, then determine an eigenvector of A belonging to 3.

$$A = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 5 \\ 1 & 8 & 4 \end{pmatrix}$$

6. MT'10 We know that $\lambda = 3$ is an eigenvalue of the matrix A below.
(a) Determine the value of the parameter p.
(b) Determine an eigenvector of the matrix A.

$$A = \begin{pmatrix} 4 & 0 & p \\ 5 & 7 & 7 \\ 1 & 1 & 5 \end{pmatrix}$$

7. MT+'14 a) Is the vector $\underline{v}$ below an eigenvector of the matrix A below?
b) Determine one eigenvalue of the matrix A, and all the eigenvectors belonging to this eigenvalue.

$$\underline{v} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \quad A = \begin{pmatrix} 4 & 3 & -5 \\ -2 & -3 & 10 \\ 1 & 3 & -2 \end{pmatrix}$$

8. MT+'15 Determine the value of the parameter p if we know that the vector $\underline{v}$ below is an eigenvalue of the matrix A below. Determine all the eigenvalues and eigenvectors of the matrix A as well.

$$A = \begin{pmatrix} 6 & p \\ 9 & 6 \end{pmatrix}, \quad \underline{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

9. Show that a square matrix is invertible if and only if 0 is not an eigenvalue of it.

10. MT'07 Let $f : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation, furthermore let $B = \{\underline{b}_1, \underline{b}_2, \underline{b}_3\}$ be a basis of $\mathbf{R}^3$. Suppose that $\underline{b}_1$ is an eigenvector that belongs to the eigenvalue $\lambda_1 = 1$, $\underline{b}_2$ is an eigenvector that belongs to the eigenvalue $\lambda_2 = 2$, and $\underline{b}_3$ is an eigenvector that belongs to the eigenvalue $\lambda_3 = 3$ of f. Determine the matrix of the linear transformation f^2 with respect to the basis B. (f^2 is defined by $f^2(\underline{v}) = f(f(\underline{v}))$.)