Introduction to the Theory of Computing 1.

Exercise-set 1.

1. a) How many positive divisors does 8800 have?
   b) How many common positive divisors do 8800 and 99000 have?

2. How many integers are there between 1 and 1000 which have an equal number of even and odd divisors?

3. a) Show that if $2^n - 1$ is prime for an integer $n \geq 1$, then n is also a prime.
   b) Show that if $2^n + 1$ is prime for an integer $n \geq 1$, then n isa power of 2.

4. Determine whether the following statements are true or not:
   a) $100 \equiv 43 \pmod{19}$,
   b) $50 \equiv -15 \pmod{13}$,
   c) $10000 \equiv 4300 \pmod{19}$.

5. For which positive integers do the following statements hold?
   a) $149 \equiv 139 \pmod{m}$,
   b) $13 \equiv 613 \pmod{m}$ and $23 \equiv 617 \pmod{m}$,
   c) $7m + 61 \equiv 4m + 76 \pmod{m}$.

6. (MT+'08) Determine all the integers n for which $3n + 1 \equiv 6 \pmod{2n}$ holds.

7. If we divide 11275 and 12299 by the same three-digit integer, we get the same remainder. What is this remainder?

8. (MT++'14) Determine whether the following statements are true for all integers n or not:
   a) If $n^2 \equiv 1 \pmod{39}$ then $n \equiv 1 \pmod{39}$ or $n \equiv -1 \pmod{39}$.
   b) If $n^2 \equiv 1 \pmod{39}$ then $n \equiv 1 \pmod{13}$ or $n \equiv -1 \pmod{13}$.

9. Determine the remainder we get if we divide
   a) 70^{70} by 23,
   b) 55^{100} by 48,
   c) 2017^{6543} by 2018,
   d) 1025^{1005} by 1023,
   e) 138^{139} by 65?

10. Determine the last two digits of the following numbers:
    a) 2001^{2017},
    b) $99^{77^{55}}$,
    c) 51^{151},
    d) $51^{151}/9$.