Hamilton-chain saturated hypergraphs

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Abstract

We say that a hypergraph $\mathcal{H}$ is hamiltonian path (cycle) saturated if $\mathcal{H}$ does not contain an open (closed) hamiltonian chain but by adding any new edge we create an open (closed) hamiltonian chain in $\mathcal{H}$. In this paper we ask about the smallest size of an r-uniform hamiltonian path (cycle) saturated hypergraph, mainly for $r = 3$. We present a construction of a family of 3-uniform path (cycle) saturated hamiltonian hypergraphs with $\Omega(n^{5/2})$ edges. On the other hand we prove that the number of edges in an r-uniform hamiltonian path (cycle) saturated hypergraph is at least $\Omega(n^{r-1})$.

Keywords: graph, saturated graph, hypergraph, hamiltonian path, hamiltonian cycle, hamiltonian chain.

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1 Introduction

Let $\mathcal{H}$ be an r-uniform hypergraph on the vertex set $V(\mathcal{H})$ =$\{v_1, v_2, \ldots, v_n\}$ where $n \geq r$. The set of the edges — r-element subsets of $V(\mathcal{H})$ — is denoted by $\mathcal{E}(H)$= $\{E_1, E_2, \ldots, E_m\}$. We will write simply V for $V(\mathcal{H})$ and $\mathcal{E}$ for $\mathcal{E}(H)$ if no confusion can arise. Denote by $\mathcal{H}(U)$ the subhypergraph of $\mathcal{H}$ induced by U, where $U \subseteq V(\mathcal{H})$.

Definition 1 *Let $\mathcal{H}$ be an r-uniform hypergraph on n vertices. An ordering $(v_1 v_2 \ldots v_{l+r-1})$ of a subset of the vertex set is called an open chain of length l between v_1 and v_{l+r-1} iff for every $i = 1, \ldots, l$ there exists an edge $E_j \in \mathcal{E}(H)$ such that $\{v_i, v_{i+1}, \ldots, v_{i+r-1}\} = E_j$. An open chain of length $n - r + 1$ is an open hamiltonian chain.*

This definition was first given in [11] and several questions on hamiltonian chains where investigated in [7, 11]. Other types of generalised cycles in hypergraphs can be found in [1, 10]. In the present paper we consider only open chains so for simplicity we will write chain instead of open chain. By removing a vertex we mean to remove also every edge containing this vertex.

Definition 2 *We say that a hypergraph $\mathcal{H}$ is hamiltonian path saturated if $\mathcal{H}$ does not contain an open hamiltonian chain but by adding any new edge we create an open hamiltonian chain in $\mathcal{H}$.*

Originally, the problem of estimating the number of edges in a hamiltonian cycle saturated graph appeared in O. Ore [12] where it is prowed that a nonhamiltonian graph (and, so, a hamiltonian cycle saturated graph) of order n has at most $\binom{n-1}{2} + 1$ edges. Bollobás [2] posed the problem of finding the minimum number, $sat(n; C_n)$, of edges in a hamiltonian cycle saturated graph on n vertices. In 1972 Bondy [3] proved that $sat(n; C_n) \geq \lceil \frac{3n}{2} \rceil$ for $n \geq 7$. Combined results of Clark, Entrigner and Shapiro [5, 4] and Xiaohui, Wenzhou, Chengxue and Yuansheng [13] show that this bound is sharp apart from a few smaller values of n. The constructions are mostly tricky graphs based on Isaacs' snarks (see [9]) and generalized Petersen graphs. It was natural to ask the same question for hamiltonian path saturated graphs. Dudek et al. [6] obtained using some modification's of Isaacs' snarks that $\lfloor \frac{3n-1}{2} \rfloor - 2 \leq sat(n; P_n) \leq \lfloor \frac{3n-1}{2} \rfloor$ for $n \geq 54$. The exact value $sat(n; P_n) = \lfloor \frac{3n-1}{2} \rfloor$ for $n \geq 54$ was determined by Frick and Singleton [8]. In the present paper we study a related problem for r-uniform hypergraphs, mainly for $r = 3$.

Definition 3 *Let $g_r(n)$ ($r \geq 2$) denote the minimum number of edges in a hamiltonian path saturated r-uniform hypergraph on n vertices.*

Hence $g_2(n) = \lfloor \frac{3n-1}{2} \rfloor$ for $n \geq 54$. On the other hand, in [11] a construction is given of an n-vertex hamiltonian path saturated r-uniform hypergraph with

$$\sim \left(\frac{1}{r!} - \frac{1}{2^r \lceil r/2 \rceil! \lfloor r/2 \rfloor!} \right) n^r$$

edges which, so far, is best known upper bound for $g_r(n)$. For $r = 3$, this yields $g_3(n) \leq \frac{5}{48} n^3 + o(n^3)$. In the present paper we improve the construction from [11] for $r = 3$. As a result, for any $n \geq 12$ we obtain a 3-uniform hypergraph with $O(n^{5/2})$ edges. It is interesting that the existence of a hamiltonian chain depends on the order of some sets in our construction. On the other hand, we obtain a general lower bound $g_r(n) \geq \binom{n}{r}/(r(n-r)+1)$ which is of order $\Omega(n^{r-1})$.

It would be desirable to generalize the result of [6] and [8] for 3-uniform hypergraphs but we have not been able to do this. The main difficulty in carrying out this construction is the fact that we do not know how to generalize Isaacs' graphs. On the other hand our construction can be seen as a generalization of Zelinka's construction [14] which is a union of $p+2$ disjoint cliques plus p vertices connected to all vertices.

2 Lower bound

Theorem 1 *If $\mathcal{H}$ an r-uniform hypergraph is hamiltonian path saturated, then $|\mathcal{E}(H)| \geq \binom{n}{r}/(r(n-r)+1)$.*

Proof. We prove that every r-tuple $E_0 = \{v_1, \ldots, v_r\}$ contains an $(r-1)$-element subset, which is contained by an edge of $\mathcal{H}$.

If $E_0 \in \mathcal{E}(H)$ then any $(r-1)$-element subset is contained by E_0 which is an edge, so the claim holds.

Now suppose that $E_0 \notin \mathcal{E}(H)$. Since $\mathcal{H}$ is hamiltonian path saturated, it does not contain a hamiltonian chain, but adding E_0 creates one. Therefore E_0 must be an edge of this hamiltonian chain, so it has a neighboring edge in the chain (even if it is at the end of the chain). This edge satisfies the conditions of the claim.

Using the claim we obtain that for all possible r-tuples we can find an edge that intersects the r-tuple in at least $(r-1)$ elements. However, in this way every such edge is counted $r(n-r)+1$ times. $\square$

3 Hamiltonian path saturated 3-uniform hypergraphs

In this section we present a construction of a family of 3-uniform hamiltonian path saturated hypergraphs. We start with two definitions.

Definition 4 *Let p and k be non-negative integers and $U_0, U_1, \ldots, U_k$ be pairwise disjoint sets of vertices such that $|U_0| = p$ and $|U_i| \geq 2$ for $i = 1, 2, \ldots, k$. Define the vertex set of the hypergraph $\mathcal{H} = \mathcal{H}(U_0, U_1, \ldots, U_k)$ to be $V(\mathcal{H}) = \bigcup_{i=0}^{k} U_i$. The edge set is defined such that the induced subhypergraph $\mathcal{H}(U_0 \cup U_i)$ is complete hypergraph for all $i = 1, 2, \ldots, k$. The family of all hypergraphs obtained by this construction is denoted by $\mathcal{I}(p, k)$.*

Definition 5 *Let $\mathcal{H} \in \mathcal{I}(p, k)$. An edge $E_0 = \{x, y, z\}$ where $x \in U_i$ and $y, z \in U_j$ or $x \in U_i$, $y \in U_0$ and $z \in U_j$ is called a jumping edge from U_i to U_j. The set of all jumping edges from U_i to U_j is denoted by $J_{i,j}$.*

If $E_1 \in J_{i_1,j_1}$ and $E_2 \in J_{i_2,j_2}$ then we say that jumping edges E_1, E_2 are from different sets when $j_1 \neq j_2$.

Let K_n be a complete graph on n vertices, $n \geq 2$, with vertices labeled by natural numbers $\{1, ..., n\}$. By $\overrightarrow{K_n}$ we denote the following orientation of K_n. Namely the oriented edges in $\overrightarrow{K_n}$ are of the form $(i, i+1), ..., (i, i+\lceil n/2 \rceil - 1)$ for $i = 1, ..., n$, where the numbers are understood cyclicly so $n + r = r$, if $r > 0$. The remaining edges of $\overrightarrow{K_n}$, for even n, are oriented in an arbitrary way. We write $i \prec j$ if there is an oriented edge from i to j in $\overrightarrow{K_n}$.

Definition 6 *Let $\mathcal{H}(U_0, U_1, \ldots, U_k) \in \mathcal{I}(p,k)$. Define the hypergraph $\mathcal{G} = \mathcal{G}(U_0, U_1, \ldots, U_k)$ as a hypergraph with vertex set $V(\mathcal{G}) = V(\mathcal{H})$ and edge set*

$$\mathcal{E}(\mathcal{G}) = \mathcal{E}(\mathcal{H}) \cup \{J_{i,j} : i \prec j\}.$$

The family of hypergraphs obtained by this construction is denoted by $\mathcal{J}(p,k)$.

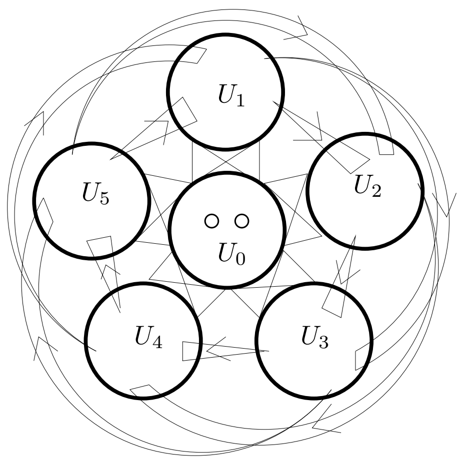

Fig. 1. A hypergraph from the family $\mathcal{J}(2,5)$.

Lemma 2 *Let $\mathcal{G} \in \mathcal{J}(p,k)$. A chain in $\mathcal{G} - U_0$ cannot contain jumping edges from two different sets.*

Proof. Suppose indirectly that a chain contains edges from two different set of jumping edges $E_1 \in J_{i_1,j_1}$ and $E_2 \in J_{i_2,j_2}$, $j_1 \neq j_2$. Without a loss of generality we can assume that there are no other jumping edges in the chain between these two edges.

By this assumption only non-jumping edges can be found between these edges on the chain. E_1 is adjacent on the chain to edges contained in U_{j_1}. These edges are adjacent to edges of the same kind and jumping edges which cannot be used now. So the chain can be continued only by such edges. However, E_2 is not adjacent to such an edge, since their intersection contain only one vertex. Therefore the chain cannot reach E_2, a contradiction. □

Theorem 3 *Let $\mathcal{G} \in \mathcal{J}(p,k)$ where p, k are non-negative integers such that $\lceil 2k/3 \rceil \geq p+2$. Let $|U_i| \in \{\alpha-1, \alpha\}$ for $i = 1, ..., k$ and $|U_j| = \alpha$ for some $j \in \{1, ..., k\}$, where α is an integer satisfying $\alpha \geq 5(p+1)+1$. Then $\mathcal{G}$ has no hamiltonian chain.*

Proof. Suppose indirectly that $v_1 v_2 ... v_n$ is a hamiltonian chain in $\mathcal{G}$. By removing all vertices of U_0 the sequence $v_1 v_2 ... v_n$ falls to m parts, where $m \leq p+1$. Each part induce a chain in $\mathcal{G} - U_0$ or consists of one or two vertices. If a part contains an edge $E \in \mathcal{E}(G)$ such that $|E \cap U_i| \geq 2$ for some $i \in \{1, ..., k\}$ then by Lemma 2 every edge in this part have at least two vertices from U_i. We say that the set U_i is a *dominating set* for this part. Let x_i denote the number of vertices of the i-th part which belong to its dominating set. Consequently, let y_i denote the number of remaining vertices in the i-th part. Recall that among every three consecutive vertices of some part at least two belong to its dominating set. Hence $x_i \geq 2(y_i - 1)$ if $x_i > 0$, and $x_i + y_i \leq 2$ otherwise. Thus $x_i + y_i \leq \frac{3}{2}\alpha + 1$. Therefore

$$k(\alpha-1) < |U_1| + ... + |U_k| = \sum_{i=1}^{m}(x_i + y_i) \leq \sum_{i=1}^{m}\left(\frac{3}{2}\alpha+1\right) \leq (p+1)\left(\frac{3}{2}\alpha+1\right), \text{ hence}$$
$$\frac{2}{3}k < (p+1)\frac{\alpha+2/3}{\alpha-1} = (p+1) + (p+1)\frac{5}{3(\alpha-1)}.$$

Thus

$$p+2 \leq \left\lceil \frac{2}{3}k \right\rceil \leq \frac{2}{3}k + \frac{2}{3} < (p+1) + (p+1)\frac{5}{3(\alpha-1)} + \frac{2}{3}, \text{ hence}$$
$$1 < (p+1)\frac{5}{\alpha-1}, \text{ a contradiction.}$$

□

Theorem 4 *Let t be a nonnegative integer and let $\mathcal{G} \in \mathcal{J}(2t, 3t+2)$. Let $|U_i| \in \{\alpha-1, \alpha\}$ for $i = 1, ..., 3t+2$ and $|U_j| = \alpha$ for some $j \in \{1, ..., k\}$, where α is an integer satisfying $\alpha \geq 10t+6$. Then $\mathcal{G}$ is hamiltonian path saturated.*

Proof. Since $\left\lceil \frac{2}{3}(3t+2) \right\rceil = 2t+2$, by Theorem 3, $\mathcal{G}$ has no hamiltonian chain. We will show that adding any new edge E to $\mathcal{G}$ creates a hamiltonian chain. Let $E = \{u, v, w\}$. There are two different types of E:

Case 1. $u \in U_i, v \in U_j, w \in U_k$ with $i \neq j, i \neq k, j \neq k$; in this case we may assume that $i \prec j$ and $j \prec k$,

Case 2. $u \in U_j, v \in U_j, w \in U_k$ with $j \prec k$.

We deal with both of the cases simultaneously.

Note that for $t \geq 2$ the set $V(\overrightarrow{K}_{3t+2}) \setminus \{j, k\}$ can be decomposed into triples (a_n, b_n, c_n), $n = 1, ..., t$, such that $a_n \prec b_n$ and $a_n \prec c_n$ for every n. Indeed, for the triples we can take consecutive vertices in the sequence $k+1, k+2, k+3, ..., \widehat{j}, ..., \widehat{k}$ where the symbol $\widehat{x}$ means that x is omitted in the sequence. Let

$$\mathcal{C} \sim j, j, ..., j, u, v, w, k, k, ..., k$$

denote the sequence containing vertices u, v, w and all vertices from the sets U_j – in the positions denoted by j – and U_k – in the positions denoted by k. Note that C is a chain in $\mathcal{G} + E$. Consequently let

$$\mathcal{C}_n \sim a, b, b, a, b, b, a, ..., a, b, b, a, (b), 0, a, c, c, a, c, c, a, ..., a, c, c, a, (c),$$

$n = 1, ..., t$, denote the sequence containing one vertex from U_0 (denoted by 0) and vertices from the set $U_{a_n} \cup U_{b_n} \cup U_{c_n} \setminus \{u\}$ in the positions denoted by a, b, c, respectively. The symbol (x) means that x may or may not occur in the sequence depending on the parity of $|U_{x_n} \setminus \{u\}|$.

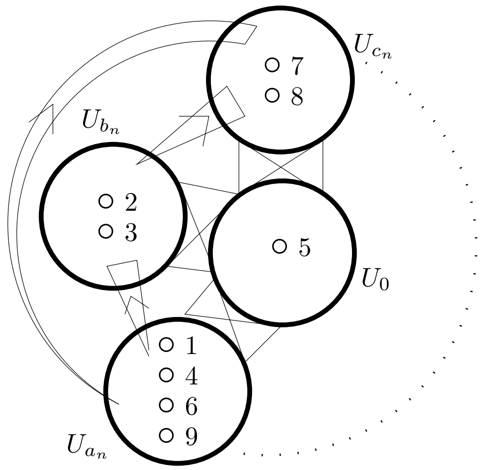

Fig. 2. The sequence 1,2,3,4,5,6,7,8,9 realises the fragment '$...a, b, b, a, 0, a, c, c, a...$' of $\mathcal{C}_n$.

Note that we are always able to place all the vertices from $U_{a_n} \cup U_{b_n} \cup U_{c_n} \setminus \{u\}$ in such sequence. Indeed, let A, B, C denote the number of a's, b's, and c's in $\mathcal{C}_n$, respectively. Then $A = \left\lfloor \frac{1}{2}B \right\rfloor + 1 + \left\lfloor \frac{1}{2}C \right\rfloor + 1$. Since $|U_{b_n}|, |U_{c_n}| \geq \alpha - 1$, $A \geq \left\lfloor \frac{\alpha-1}{2} \right\rfloor + 1 + \left\lfloor \frac{\alpha-2}{2} \right\rfloor + 1 = \alpha$ because the vertex u may belong to U_{b_n} or to U_{c_n}. If $2\alpha - 3 < B + C$ $(= |U_{b_n} \cup U_{c_n} \setminus \{u\}|)$ or $|U_{a_n} \setminus \{u\}| < \alpha$ then we can delete from $\mathcal{C}_n$ an appropriate number of a's without ruining the chain. In any case we can modify $\mathcal{C}_n$ in such a way that the resulting sequence contains exactly one vertex from U_0 and all vertices from $U_{a_n} \cup U_{b_n} \cup U_{c_n} \setminus \{u\}$. We denote such modified $\mathcal{C}_n$ by $\mathcal{C}'_n$. Clearly each $\mathcal{C}'_n$ is a chain in $\mathcal{G} + E$. The following sequence is also a chain in $\mathcal{G} + E$

$$\mathcal{C}, 0, \mathcal{C}'_1, 0, \mathcal{C}'_2, 0, ..., 0, \mathcal{C}'_t,$$

(here symbols 0 denote different vertices from the set U_0). Since $\mathcal{C}$ does not contain a vertex from U_0 and each $\mathcal{C}'_n$ contains exactly one vertex from U_0, the above sequence contains all vertices of $\mathcal{G}$, hence is a hamiltonian chain.

If $t = 1$ then, due to symmetry, we can assume that $j = 1$ and $k = 2$ or $j = 1$ and $k = 3$. In the former case we can repeat previous argument since in $V(\overrightarrow{K_5}) \setminus \{1, 2\}$, $3 \prec 4$ and $3 \prec 5$. Assume that $j = 1$ and $k = 3$. Then $i = 4$, 5 or 1 because $i \prec j$ or $i = j$. If $i = 4$ then the following sequence, or its modification resulting by deleting an appropriate number of 3's, is a hamiltonian chain in $\mathcal{G} + E$

$$1, 1, ..., 1, v, u, w, 4, 4, 3, 4, 4, 3, ..., 3, 4, 4, 3, (4), 0, 3, 5, 5, 3, 5, 5, 3, ..., 3, 5, 5, 3, (5), 0, 2, 2, ..., 2$$

(as previously, symbols x different from u, v, w denote distinct vertices from the set U_x while symbol (x) denote that x may or may not appear in the sequence depending on the parity of $|U_x|$). Indeed, let A, B, C denote the number of 3's, 4's and 5's in the sequence, respectively. Then $A = \left\lceil \frac{B}{2} \right\rceil + \left\lfloor \frac{C}{|2|} \right\rfloor + 1 \geq \left\lceil \frac{\alpha-1}{2} \right\rceil + \left\lfloor \frac{\alpha-1}{2} \right\rfloor + 1 = \alpha$. If $2\alpha - 2 < B + C$ $(= |U_4| + |U_5|)$ or $|U_3| < \alpha$ then we can delete from the sequence an appropriate number of 3's without spoiling the chain. Similar argument holds when $i = 5$ or $i = 1$.

Finally, it is clear that $\mathcal{G} + E$ contains a hamiltonian chain if $t = 0$. $\square$

Theorem 5 *For every $n \geq 12$ there exists a 3-uniform hamiltonian path saturated hypergraph with at most $\frac{3\sqrt{30}}{25}n^{5/2} + o(n^{5/2})$ edges.*

Proof. Let $t_0 := \left\lfloor \frac{\sqrt{10}}{30}\sqrt{3n+4} - \frac{2}{3} \right\rfloor$. Hence $t_0 \geq 0$. Let $\mathcal{G} \in \mathcal{J}(2t_0, 3t_0+2)$ with the property that the sets U_i have equal or nearly equal size. Hence $|U_i| \in \{\alpha - 1, \alpha\}$, $i = 1, ..., 3t_0+2$, where $\alpha = \left\lceil \frac{n-2t_0}{3t_0+2} \right\rceil$. Moreover, at least one U_j satisfies $|U_j| = \alpha$. By simple computations

$$\frac{n-2t}{3t+2} \geq 10t + 6 \Leftrightarrow t \leq \frac{\sqrt{10}}{30}\sqrt{3n+4} - \frac{2}{3}.$$

Hence α satisfies conditions from Theorem 4. Thus $\mathcal{G}$ is hamiltonian path saturated. Note that the number of edges of any hypergraph $\mathcal{G}' \in \mathcal{J}(2t, 3t+2)$ with $|U_i| \in \{\alpha - 1, \alpha\}$, $i = 1, ..., 3t+2$, satisfies

$$\begin{aligned}
|\mathcal{E}(\mathcal{G}')| &\leq \binom{\alpha+2t}{3}(3t+2) + \binom{3t+2}{2}\binom{\alpha+2t}{2}\alpha \leq \frac{(\alpha+2t)^3}{6}(3t+2) + \frac{(3t+2)^2(\alpha+2t)^2\alpha}{4} \\
&\simeq (n+6t^2+2t)^2\left(\frac{n+6t^2+2t}{6(3t+2)^2} + \frac{n-2t}{4(3t+2)}\right). \qquad (1)
\end{aligned}$$

Hence for $t = t_0$

$$|\mathcal{E}(\mathcal{G})| \leq \left(n + 6\frac{3n+4}{90}\right)^2 \frac{n}{12\frac{\sqrt{10}}{30}\sqrt{3n+4}} + o(n^{5/2}) = \frac{3\sqrt{30}}{25}n^{5/2} + o(n^{5/2}).$$

□

4 Concluding remarks

We have constructed a family of 3-uniform hamiltonian chain saturated hypergraphs. The main result is Theorem 5, which gives the hypergraphs with the smallest number of edges. Since the lower bound we gave is smaller than the number of edges in the construction by a factor $n^{1/2}$ the question is still open. We conjecture that there exists an r-uniform hamiltonian path saturated hypergraph with $O(n^{r-1})$ edges.

Note that our construction cannot be improved by taking another t. Indeed, if we take t of order different from $n^{1/2}$ then, by (1), $|\mathcal{E}(\mathcal{G})|$ is asymptotically greater than the value obtained in Theorem 5. Hence t of the form $a\sqrt{n}$ is best. Then

$$|\mathcal{E}(\mathcal{G})| \sim (n + 6a^2n)^2 \frac{n}{12a\sqrt{n}} + o(n^{5/2}) = \frac{\left(1+6a^2\right)^2}{12a}n^{5/2} + o(n^{5/2}).$$

Recall that $t < \frac{\sqrt{10}}{30}\sqrt{3n+4} - \frac{2}{3} \sim \frac{1}{\sqrt{30}}n^{1/2}$. On the other hand it is easy to check that the function $f(a) = \frac{\left(1+6a^2\right)^2}{12a}$ is decreasing for $a \in (0, 1/\sqrt{18})$. Thus taking the largest possible value of t gives best result.

We observe that the same bounds can be obtained in case we consider closed hamiltonian chain $v_1, v_2, ...v_n v_1 v_2$ (hamiltonian cycle) instead of an open one $v_1, v_2, ...v_n$. The proof of the lower bound is very similar to the proof of Theorem 1. On the other hand the upper bound can be realized by a hypergraph $\mathcal{G} \in \mathcal{J}(2t+1, 3t+2)$ with $\alpha \geq 10t + 6$.

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References

[1] J. C. Bermond, A. Germa, M. C. Heydemann and D. Sotteau, *Hypergraph Hamiltoniens*, Problèmes combinatores et théorie des graphes, Orsey (1976) No. 260 39–43.

[2] B. Bollobás, *Extremal Graph Theory*, Academic Press, New York (1978).

[3] J. A. Bondy, *Variations on the hamiltonian theme*, Canad. Math. Bull. **15** (1972) 57–62.

[4] L. H. Clark, R. C. Entringer, *Smallest maximally nonhamiltonian graphs*, Period. Math. Hung. **14** (1983), 57–68.

[5] L. H. Clark, R. C. Entringer, H. D. Shapiro, *Smallest maximally nonhamiltonian graphs II*, Graphs and Combin. **8** (1992) 225–231.

[6] A. Dudek, G. Y. Katona, A. P. Wojda, *Hamiltonian path saturated graphs with small size*, Discr. Appl. Math. 154 (2006) 1372–1379.

[7] P. Frankl, G.Y. Katona, *Extremal k-edge-hamiltonian hypergraphs*, Discrete Math. 308 (2008), no. 8, 1415–1424.

[8] M. Frick, J. Singleton, *Lower bound for the size of maximal nontraceable graphs*, Electronic J. Combin. 12 (2005) R32.

[9] R. Isaacs, *Infinite families of nontrywial trivalent graphs which are not Tait colorable*, Amer. Math. Mon. **82** (1975) 221–239.

[10] D. Khn, D. Osthus,*Loose Hamilton cycles in 3-uniform hypergraphs of high minimum degree*, J. Combin. Theory Ser. B 96 (2006), no. 6, 767–821.

[11] G.Y. Katona, H. Kierstead, *Hamiltonian chains in hypergraphs*, J. Graph Theory **30** (1999) 205–212.

[12] O. Ore, *Arc covering of graphs*, Ann. Mat. Pura Appl. **55** (1961) 315–321.

[13] L. Xiaohui, J. Wenzhou, Z. Chengxue, Y. Yuansheng, *On smallest maximally nonhamiltonian graphs*, Ars Combin. **45** (1997) 263–270.

[14] B. Zelinka, *Graphs maximal with respect to absence of hamiltonian path*, Discuss. Math. Graph Theory **18** (1998) 205–208.