Transformations of context-free grammars, pumping lemma

1. Remove the useless symbol from the given context free grammar

$$\begin{aligned} S &\longrightarrow \mathtt{a}B \mid \mathtt{b}X \\ A &\longrightarrow B\mathtt{ac} \mid \mathtt{b}SX \mid \mathtt{a} \\ B &\longrightarrow \mathtt{a}SB \mid \mathtt{b}BX \\ X &\longrightarrow SBD \mid \mathtt{a}B\mathtt{a} \mid \mathtt{ac} \end{aligned}$$

2. Remove the useless symbol from the given context free grammar

$$\begin{aligned} A &\longrightarrow \mathtt{abc} \mid B\mathtt{bcc} \\ B &\longrightarrow B\mathtt{c} \mid \mathtt{a}C\mathtt{c} \\ C &\longrightarrow \mathtt{b}C\mathtt{b} \mid B\mathtt{c} \\ D &\longrightarrow D\mathtt{b} \mid \mathtt{c} \end{aligned}$$

3. Construct an equivalent grammar without chain rules and useless symbols from the following grammar

$$\begin{aligned} S &\longrightarrow AB \\ A &\longrightarrow \mathtt{a} \\ B &\longrightarrow C \mid \mathtt{b} \\ C &\longrightarrow D \\ D &\longrightarrow E \\ E &\longrightarrow \mathtt{a} \end{aligned}$$

4. Construct an eqivalent grammar without ε-rules, chain rules and useless symbols

$$\begin{aligned} A &\longrightarrow ABA \mid C \mid \varepsilon \\ B &\longrightarrow AC \mid \mathtt{a} \mid BD \\ C &\longrightarrow C\mathtt{b} \mid \mathtt{a} \mid \varepsilon \\ D &\longrightarrow \mathtt{a}CC \mid \mathtt{b} \end{aligned}$$

5. By the pumping lemma show that the following languages is not context free

$$L = \{\mathtt{0}^k\mathtt{1}0^k\mathtt{1}0^k : k \geq 0\}$$

6. Is the language $L = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k : i < j < k\}$
   a) regular?
   b) context free?
   c) not context free?

7. For any two languages L_1 and L_2 let $L_1 \oplus L_2$ denote the lanuage that consists of all words that belong to exactly one of L_1 and L_2.

   Show that there exist L_1 and L_2 context free, such that
   a) L is context free
   b) L is not context free